Math 210 Calculus II **May 6, 1998**

Final Exam

1. (20 points) TRUE or FALSE.

   (A) A particle moving through space with constant speed has zero acceleration.

   (B) If $f(x,y)$ has continuous second-order partials on $\mathbb{R}^2$ and if $\nabla f(x,y) = P(x,y)\,\mathbf{i} + Q(x,y)\,\mathbf{j}$, then $P_y = Q_x$.

   (C) If $\mathbf{F}$ is a continuous vector field on $\mathbb{R}^2$ then the line integral of $\mathbf{F}$ around any closed path is zero.

   (D) A function $f(x,y)$ can be continuous at the point (x_0, y_0) even though it is not differentiable at (x_0, y_0).

   (E) If C is a smooth curve given by the vector function $\mathbf{r}(t)$, $a \le t \le b$, and $f(x,y)$ is a differentiable function whose gradient vector ∇f is continuous, then

$$\int_C \nabla f \cdot d\,\mathbf{r} = f(\,\mathbf{r}(b)) - f(\,\mathbf{r}(a)).$$

2. (20 points) Suppose that $f(1,3) = 4$, $\dfrac{\partial f}{\partial x}(1,3) = 2$, and $\dfrac{\partial f}{\partial y}(1,3) = 3$. Use the differential approximation to estimate $f(1.1, 2.9)$.

3. (20 points) Find an equation for the tangent plane to the graph of the function $f(x,y) = \ln(2x+y)$ at the point $(-1,3,0)$.

4. (20 points) Find the critical points of the function $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$ and determine which are maxima, minima or saddles.

5. (20 points) Find the maximum value of the function $f(x,y,z) = x - y + 3z$ on the ellipsoid $x^2 + y^2 + 4z^2 = 4$.

6. (20 points) Change to polar coordinates and evaluate $\displaystyle\iint_{D_R} e^{-x^2-y^2}\,dA$ where D_R is the disk bounded by the circle of radius R centered at the origin. ($D_R = \{(x,y)\colon x^2+y^2 \le R^2\}$).

7. (20 points) Change the order of integration and evaluate the iterated integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dx\,dy.$$

PROBLEMS 8 THROUGH 10 ARE ON THE BACK

8. (20 points) Find the volume of the region bounded by the two paraboloids $z = x^2 + y^2$ and $z = 18 - x^2 - y^2$.

9. (20 points) For each vector field below either find a function whose gradient is the given field (i.e. a potential function), or explain why no such function exists.

   (a) $(e^{2x} + x \sin y)\,\mathbf{i} + (x^2 \cos y)\,\mathbf{j}$

   (b) $(ye^x + \sin y)\,\mathbf{i} + (e^x + x\cos y)\,\mathbf{j}$

10. (20 points) Use Green's Theorem to evaluate the line integral $\oint_C xy\,dx + y^5\,dy$, where C is the triangle with vertices $(0,0)$, $(2,0)$, and $(2,1)$, oriented in the counterclockwise (positive) direction.