Math 210 **Final Exam** Fall 2005

NAME: ______________________________

You must hand this sheet in with your exam in order to receive a grade.

Calculators are not allowed on this exam.

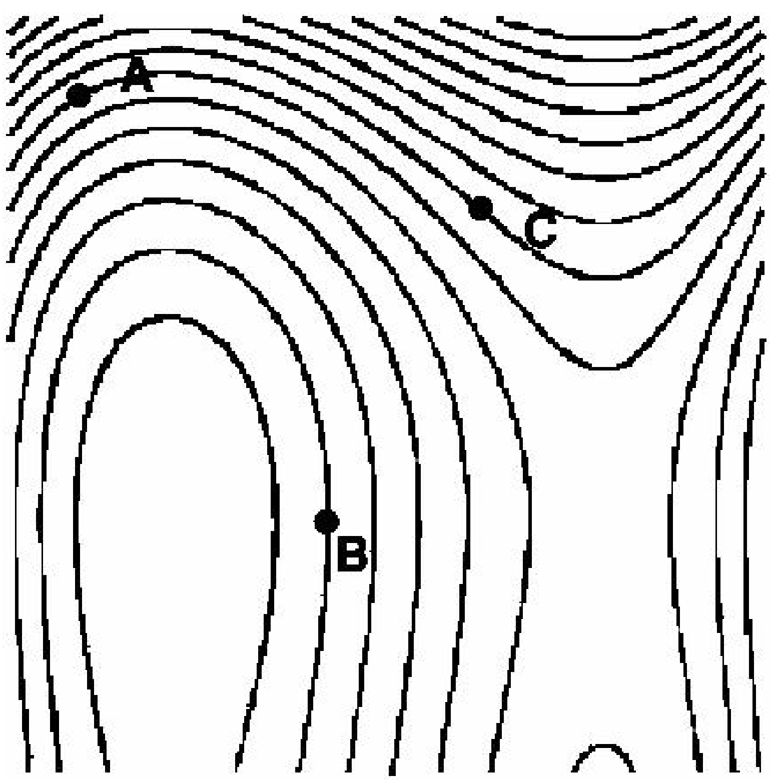
1. (20 points) For the curve $\mathbf{r}(t) = \langle 3\sin t, -3\cos t, 4t\rangle$
   (a) Find the velocity $\mathbf{v}(t)$ and the acceleration $\mathbf{a}(t)$.
   (b) Find the length of the curve where $0 \le t \le \pi$.

2. (20 points) Find the directional derivative of the function

$$f(x, y, z) = (y + \sin(z + xy^2))^2$$

   at the point $(0, 1, 0)$ in the direction towards the point $(4, 0, 3)$.

3. (20 points) The contour plot below shows a region containing *one local maximum* and *one saddle* for a function $f(x, y)$. For each of the three marked points A, B, C, draw an arrow representing the gradient vector ∇f at that point. Make sure that the arrows clearly indicate both the directions and the relative sizes of the gradient vectors at the three points.



4. (20 points) Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = 2x^2 + 3y^2$ on the unit circle $x^2 + y^2 = 1$.

5. (20 points) Evaluate the iterated integral by first reversing the order of integration.

$$\int_0^2 \int_{x^2}^{2x} f(x,y)\ dydx$$

6. (20 points) Let E be the solid bounded by the paraboloid $z = 4x^2 + 4y^2$ and the plane $z = 4$. Compute the volume of E.

7. (20 points) Evaluate

$$\iiint\limits_R \frac{dV}{x^2 + y^2 + z^2}$$

where R is the region between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$.

8. (20 points) Compute the line integral $\int_C y\,dx - x\,dy$ where C is the upper semicircle of radius 1 oriented from $(1,0)$ to $(-1,0)$ (i.e. counter-clockwise).

9. (20 points) Consider the vector field $\mathbf{F}(x,y) =< 1 + 2ye^{2x}, 2y + e^{2x} >$
   (a) Show that $\mathbf{F}$ is conservative.
   (b) Find a potential function f such that $\mathbf{F} = \nabla f$.
   (c) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve C given by $\mathbf{r}(t) = \langle t, 1 + \cos(\pi t)\rangle$ for $0 \leq t \leq 1$.

10. (20 points) Use Green's Theorem to evaluate

$$\oint_C (3x + 2y)\ dx + (y^2 - x^2)\ dy$$

where C is the triangle with vertices $(-1,0)$, $(1,0)$ and $(0,1)$ traversed in that order.