Combinatorics Seminar

Grid Ramsey problem and related questions

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Abstract: The Hales–Jewett theorem is one of the pillars of Ramsey theory, from which many other results follow. A celebrated theorem of Shelah says that Hales–Jewett numbers are primitive recursive. A key tool used in his proof, now known as the cube lemma, has become famous in its own right. In its simplest form, this lemma says that if we color the edges of the Cartesian product $K_n \times K_n$ in r colors then, for n sufficiently large, there is a rectangle with both pairs of opposite edges receiving the same color. Shelah's proof shows that $n = r^{\binom{r+1}{2}} + 1$ suffices, and more than twenty years ago, Graham, Rothschild and Spencer asked whether this bound can be improved to a polynomial in r. We show that this is not possible by providing a superpolynomial lower bound in r. We will also discuss a deep connection between this problem and generalized Ramsey numbers, and present a solution to a problem of Erd\H{o}s and Gy\'arf\'as on the transition of asymptotics of generalized Ramsey numbers.

Joint work with David Conlon (Oxford), Jacob Fox (MIT), and Benny Sudakov (ETH Zurich)

Monday, October 20 at 3:00 PM in SEO 427