Mathematics, Statistics, and Computer Science **@ UIC**

Departmental Colloquium

Roots, Schottky semigroups, and a proof of Bandt's Conjecture

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Abstract: In 1985, Barnsley and Harrington defined a ''Mandelbrot Set'' M for pairs of similarities - this is the set of complex numbers z with norm less than 1 for which the limit set of the semigroup generated by the similarities $x \to zx$ and $x \to z(x-1)+1$ is connected. Equivalently, M is the closure of the set of roots of polynomials with coefficients in $\{-1,0,1\}$. Barnsley and Harrington already noted the (numerically apparent) existence of infinitely many small ''holes'' in M, and conjectured that these holes were genuine. These holes are very interesting, since they are ''exotic'' components of the space of (2 generator) Schottky semigroups. The existence of at least one hole was rigorously confirmed by Bandt in 2002, but his methods were not strong enough to show the existence of infinitely many holes; one difficulty with his approach was that he was not able to understand the interior points of M, and on the basis of numerical evidence he conjectured that the interior points are dense away from the real axis. We introduce the technique of *traps* to construct and certify interior points of M, and use them to prove Bandt's Conjecture. Furthermore, our techniques let us certify the existence of infinitely many holes in M. This is joint work with Sarah Koch and Alden Walker.

Friday, October 3 at 3:00 PM in SEO 636