Mathematics, Statistics, and Computer Science **@ UIC**

Graduate Number Theory Seminar

Most Subsets are Balanced in Finite Groups

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Abstract: The sumset is one of the most basic and central objects in additive number theory. Many of the most important problems (such as Goldbach's conjecture and Fermat's Last theorem) can be formulated in terms of the sumset $S + S = \{x + y : x, y \in S\}$ of a set of integers S. A finite set of integers A is sum-dominated if $|A + A| > |A - A|$. Though it was believed that the percentage of subsets of $\{0, \dots, n\}$ that are sum-dominated tends to zero, in 2006 Martin and O'Bryant proved a very small positive percentage are sum-dominated if the sets are chosen uniformly at random (through work of Zhao we know this percentage is approximately $4.5 \cdot 10^{-4}$). While most sets are difference-dominated in the integer case, this is not the case when we take subsets of many finite groups. We show that if we take subsets of larger and larger finite groups uniformly at random, then not only does the probability of a set being sum-dominated tend to zero but the probability that $|A + A| = |A - A|$ tends to one, and hence a typical set is balanced in this case.

Monday, October 20 at 3:00 PM in SEO 712