Mathematics, Statistics, and Computer Science **@ UIC**

Geometry, Topology and Dynamics Seminar

Unimodal Maps and Inverse Limit Spaces

Lori Alvin (University of Denver)

Abstract: In this talk we investigate some of the continua (compact, connected metric spaces) that occur as inverse limit spaces of unimodal bonding maps. The inverse limit space of a single map $f : I \to I$ is

$$\lim_{\longleftarrow} f = \{x = (x_0, x_1, x_2, \ldots) : x_n \in I \text{ and } f(x_{n+1}) = x_n \text{ for all } n \in \mathbb{N}\}$$

and has metric $d(x, y) = \sum_{i=0}^{\infty} \frac{|x_i - y_i|}{2^i}$. We begin by exploring the continua that arise as inverse limit spaces from a single logistic map of the form $g_a(x) = ax(1-x)$, where $a \in [0, 4]$. We are particularly interested in drawing the inverse limits that arise from the family of logistic maps seen within the classical period doubling bifurcation diagram. We then use the period doubling bifurcation to gain an intuition for the inverse limit space of the logistic map with parameter $a \approx 3.569945668$, also called the Feigenbaum limit. This map, sometimes referred to as the 2^∞ map, is the unique logistic map with period points of period 2^n for all $n \in \mathbb{N}$ and no other periodic points; the action of $g|_{\omega(c,g)}$ is topologically conjugate to a special type of map called an adding machine or odometer. We conclude by discussing inverse limit spaces whose single bonding maps have embedded adding machines.

Monday, March 30 at 3:00 PM in SEO 636