Combinatorics Seminar

Largest union-intersecting families

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Abstract: Janos Korner asked the following question. Let $[n] = \{1, 2, \ldots, n\}$ and let $\mathcal{F} \subset 2^{[n]}$ be a family of its subsets. It is called union-intersecting if $(F_1 \cup F_2) \cap (F_3 \cup F_4)$ is non-empty whenever $F_1, F_2, F_3, F_4 \in \mathcal{F}$ and $F_1 \neq F_2, F_3 \neq F_4$. What is the maximum size of a union-intersecting family? This question is answered in the present paper. The optimal construction when n is odd consists of all subsets of size at least $\frac{n-1}{2}$ while in the case of even n it consists of all sets of size at least $\frac{n}{2}$ and sets of size $\frac{n}{2} - 1$ containing a fixed element, say 1. We also proved some extensions, variants and analogues of this statement. The following one is an example. Suppose that $\mathcal{F}$ is a union-intersecting family of k-element subsets of $[n]$. We found that the optimal construction for this problem consists of all k-element subsets of size k containing the element 1, and one more additional set, for $n > n(k)$. The results were jointly achieved with Daniel T. Nagy.

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