Analysis and Applied Mathematics Seminar

Sharp interface limit in a phase field model of cell motility

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Abstract: We study the motion of a eukaryotic cell on a substrate and investigate the dependence of this motion on key physical parameters such as strength of protrusion by actin filaments and adhesion. This motion is modeled by a system of two PDEs consisting of the Allen-Cahn equation for the scalar phase field function coupled with a vectorial parabolic equation for the orientation of the actin filament network. The two key properties of this system are (i) presence of gradients in the coupling terms and (ii) mass (volume) preservation constraints. We pass to the sharp interface limit to derive the equation of the motion of the cell boundary, which is mean curvature motion perturbed by a novel nonlinear term. We establish the existence of two distinct regimes of the physical parameters. In the subcritical regime, the well-posedness of the problem is proved (M. Mizuhara et al., 2015). Our main focus is the supercritical regime where we established surprising features of the motion of the interface such as discontinuities of velocities and hysteresis in the 1D model, and instability of the circular shape and rise of asymmetry in the 2D model. Because of properties (i)-(ii), classical comparison principle techniques do not apply to this system. Furthermore, the system can not be written in a form of gradient flow, which is why Γ -convergence techniques also can not be used. This is joint work with V. Rybalko and M. Potomkin.

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