

Logic Seminar

Shelah's eventual categoricity conjecture in universal classes

Sebastien Vasey (CMU)

Abstract: Abstract elementary classes (AECs) are an axiomatic framework encompassing classes of models of an $L_{\lambda,\omega}$ sentence, as well as numerous algebraic examples. They were introduced by Saharon Shelah in the mid seventies. One of Shelah's goals was to study generalizations of Morley's categoricity theorem to the infinitary setup. Among several variations, Shelah conjectured the following eventual version: An AEC categorical in a high-enough cardinal is categorical on a tail of cardinals.

In this talk, we will prove the conjecture for universal classes. It is an interesting type of AEC introduced by Shelah in a milestone 1987 paper [Sh:300] (the work was done in 1985). They correspond approximately to classes of models of a universal $L_{\lambda,\omega}$ sentence. The proof of the conjecture proceeds by first observing that any universal class satisfies tameness: a locality property isolated by Grossberg and VanDieren which says that orbital types are determined by their small restrictions. Next, several structural properties are derived from categoricity: the class has amalgamation on a tail and in fact admits a well-behaved forking-like independence relation. Finally, a definition of a unidimensionality-like property (due to Shelah) is shown to follow from categoricity in a single cardinal and imply categoricity on a tail of cardinals. The argument generalizes to tame AECS which have primes over sets of the form M_a .

Tuesday, October 20 at 4:00 PM in SEO 427
