Mathematics, Statistics, and Computer Science **@ UIC**

Dissertation Defense

Dynamics of Equicontinuous Group Actions on Cantor Sets

Jessica Dyer (University of Illinois, Chicago)

Abstract: A Vietoris solenoid is the inverse limit of $n - to - 1$ covering maps over the torus, which are all homogeneous.McCord introduced generalized weak solenoids and showed that they are homogeneous if the monodromy action is defined by a normal chain. Schori showed by construction that non-homogeneous weak solenoids exist. Rogers and Tollefson showed that there exists a weak solenoid that is not defined by a normal chain, but is homogeneous. They also constructed a non-homogeneous solenoid given by covering maps which are regular from level i to $i - 1$, but whose composition onto the base space is non-regular. Fokkink and Oversteegen gave a criterion in terms of defining group chains for a weak solenoid to be homogeneous, i.e. for the monodromy action to be regular. In this work, we investigate further the properties of the dynamics of group chains. We show that each group chain yields a minimal equicontinuous Cantor dynamical system. Conversely, we use a method of Kakutani-Rokhlin partitions to show that minimal equicontinuous Cantor dynamical systems can be represented by group chains. We then use their associated chains to classify minimal equicontinuous Cantor dynamical systems as regular, weakly regular, or irregular. We show that this classification is an invariant of the cardinality of the set of orbits of the Automorphism group. We consider the set $\mathfrak{G}_\phi$ of all group chains associated to a dynamical system, and show that the classification as regular, weakly regular, or irregular is an invariant of the number of equivalence classes of chains in $\mathfrak{G}_\phi$. We introduce a new invariant of a dynamical system called the *discriminant group*, and show that

Tuesday, October 20 at 2:00 PM in SEO 1227

its cardinality is related to the degree of non-homogeneity of the system. We give new proofs using group chains of the irregularity of the Schori and Rogers and Tollefson solenoids, and we introduce new examples of group chains which are weakly regular and have either finite or infinite discriminant group.

Tuesday, October 20 at 2:00 PM in SEO 1227