Statistics and Data Science Seminar

Overlaps and Pathwise Localization in the Anderson Polymer Model

Mike Cranston (UCI)

Abstract: We consider large time behavior of typical paths under the Anderson polymer measure. If $\mathsf{P}^{\mathsf{x}}_{\kappa}$ is the measure induced by rate κ, simple, symmetric random walk on $\mathbb{Z}^{\mathsf{d}}$ started at x, this measure is defined as \[d\mu^x_{\kappa,\beta,T}T(X)={ 1} \exp\left\{\beta\int_0^T dW_{X(s)}(s)\right\}dP^x_\kappa(X)\] where $\{\mathsf{W}_{\mathsf{x}} : \mathsf{x} \in \mathbb{Z}^{\mathsf{d}}\}$ is a field of iid standard, one-dimensional Brownian motions, $\beta > 0, \kappa > 0$ and $\mathsf{Z}_{\kappa,\beta,\mathsf{t}}(\mathsf{x})$ the normalizing constant. We establish that the polymer measure gives a macroscopic mass to a small neighborhood of a typical path as $\mathsf{T} \to \infty$, for parameter values outside the perturbative regime of the random walk, giving a pathwise approach to polymer localization, in contrast with existing results. The localization becomes complete as $\frac{\beta^2}{\kappa} \to \infty$ in the sense that the mass grows to 1. The proof makes use of the overlap between two independent samples drawn under the Gibbs measure $\mu^{\mathsf{x}}_{\kappa,\beta,\mathsf{T}}$, which can be estimated by the integration by parts formula for the Gaussian environment. Conditioning this measure on the number of jumps, we obtain a canonical measure which already shows scaling properties, thermodynamic limits, and decoupling of the parameters. This talk is based on joint work with Francis Comets.

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