

Geometry, Topology and Dynamics Seminar

Molino theory for laminations

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Abstract: A foliation of a compact manifold can be considered as a generalized dynamical system, in the sense of Smale. The study of the dynamical properties of foliations has been an active area of research for the past 40 years. A smooth foliation is Riemannian, if the normal bundle of the foliation admits a Riemannian metric invariant under the action of the holonomy pseudogroup of the foliation. Riemannian foliations are very rigid geometric structures, and they are completely classified by Molino theory.

Ghys asked in 1988 whether Molino theory can be generalized to a topological setting. In this setting, one considers foliations of compact topological spaces, which do not admit normal bundles, and where the transversals need not be locally connected. The condition analogous to the existence and invariance of a Riemannian metric in this non-differentiable setting, is the assumption of equicontinuity of the holonomy pseudogroup of the foliation. Alvarez Lopez, Candel, and Moreira Galicia gave a version of a Molino-like theory for foliated spaces under the additional assumption that the closure of the holonomy pseudogroup is strongly quasi-analytic, that is, it satisfies the condition of local generation.

In this talk, we consider foliated spaces with totally disconnected transversals, which we call matchbox manifolds, and use

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the methods of topological dynamics and continuum theory to develop a Molino-like classification of all such spaces. We show that for matchbox manifolds, the Molino sequence need not be well-defined, and specify the conditions under which it is well-defined. We outline the classes of matchbox manifolds, for which the local generation condition holds or does not hold, and study other properties of these spaces. Inspired by the result of Lubotzky about the existence of torsion in profinite completions of torsion-free groups, we construct a class of examples with well-defined non-trivial Molino sequences, where the non-triviality of the Molino sequence cannot be explained by the holonomy properties of leaves in the matchbox manifold. The examples that we construct and study show that this class of dynamical systems is far from being completely classified.

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