Mathematics, Statistics, and Computer Science   **@ UIC**

Logic Seminar

Accessible categories and abstract elementary classes

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Abstract: Bootstrapping structural properties, via accessible images

We discuss recent joint work with J. Rosicky, [LR], involving new applications of an as-yet-underappreciated tool for the analysis of categories of structures arising in abstract model theory, namely that, under the assumption of sufficiently strongly compact cardinals, the (powerful) image of any accessible functor is accessible ([MP], refined in [BrR]). Although some of these applications are technical (tameness of abstract elementary classes, i.e. AECs, in [LR16]; strong metric tameness of metric AECs in [LR]), we focus on two very simple ones: the amalgamation and joint embedding properties. The basic insight is that each property amounts to a question of the following form: given a diagram of shape A in category of structures K, can it be completed to a diagram in K of shape A'?

We can then rephrase this problem in terms of the forgetful functor U:K^{A'}—>K^A, whose (powerful) image consists precisely of the completable diagrams. As U is accessible, the theorem of [BrR] implies that this image is k-accessible for sufficiently strongly compact k, from which it follows that completability of diagrams involving objects of size up to k implies the completability of diagrams of objects of arbitrary size. That is, this is precisely what one needs to bound the Hanf

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numbers for amalgamation and joint embedding. This generalizes the results of [BaBo] from AECs to general accessible categories, thereby encompassing e.g. metric AECs, mu-AECs, and so on, with the added benefit of being almost purely visual, replacing delicate syntactic manipulations with simple questions about diagrams in well-behaved categories.

[BaBo] Baldwin, John, and Will Boney. Hanf numbers and presentation theorems in AECs. To appear in Beyond First Order Model Theory, CRC Press. [BrR] Brooke-Taylor, Andrew, and Jiri Rosicky. Accessible images revisited. Submitted, arXiv:1506.01986. [LR] Lieberman, Michael and Jiri Rosicky. Bootstrapping structural properties, via accessible images. Submitted, arXiv: 1610.07816. [LR16] ————————-. Classification theory for accessible categories. Journal of Symbolic Logic, 81(1): 3048-3066 (2016). [MP] Makkai, Michael and Robert Par\' e. Accessible Categories: The Foundations of Categorical Model Theory. AMS 1989.

Mike works at the intersection of category theory and model theory. He will be around all of next week.

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