Careers in Mathematics
Math Club – September 24, 2008

Want a Career in Mathematics?

There are lots of options! To quote from the AMS essay "What Mathematicians Do",

> Mathematicians make it possible to send secure emails and buy things online. Mathematicians are essential to analyze data and design accurate models in fields as diverse as biology and finance. Mathematicians enabled researchers to complete The Human Genome Project quickly. And because of the prevalence of the computer at work and at play, mathematicians will continue to touch everyone in modern society. [http://www.ams.org/employment/whatmathematiciansdo.html]

You can find articles and information about Careers in Mathematics & Statistics here

- http://www.ams.org/employment/undergrad.html
- http://www.ams.org/employment/undergrad.html#careers

Why Grad School?

An advanced degree in Mathematics, Statistics or Computer Science is the key to a wide variety of career options. There are three types of advanced degrees: Masters (MS), Doctorate (PhD or DA), and Masters of Science Teaching (MST).

A Masters Degree usually requires two additional years of advanced courses in your area of interest, such as Applied Math, Pure Math, Statistics, BioStatistics, and Computer Science. At UIC, these courses are at the 400 and 500 levels. You must also pass the Masters Exam, or complete a Masters Thesis.

The Masters Degree opens up many doors to careers in Mathematics. For example, graduates with a Masters in Statistics find employment in industry or government, where statisticians are in demand. A Masters Degree in Applied Math or Computer Science is valuable when applying for a job in industry.

The Masters Degree is required for faculty at most Junior Colleges, such as the Chicago City Colleges, or regional schools such as Moraine Junior College and Triton Junior College, to list a few schools where MSCS graduates have found employment.

The Masters Degree can also be the stepping stone towards the doctorate. Most doctorate programs require the Masters Degree before admission to the doctorate program.

The Masters of Teaching Degree has the goal of training the student for teaching at either the elementary level (MST-elem) or secondary level (MST-sec). These are more specialized degrees, in that their curriculum is a mix of learning subject matter in math, as well as in education and teaching methods.

The Doctoral Degree is required for becoming a tenured faculty member at most four-year colleges and universities. The Doctoral Degree requires typically two to three years of advanced courses past the undergraduate degree, plus conducting a research program in some topic of specialization, leading towards your doctoral thesis. For some, the doctoral thesis is like taking a pleasant walk (the course work) and then discovering a cliff face in front of you which you must scale. For others, research in some field of mathematics "just happens". Whichever it turns out for you, the doctorate is the rite-of-passage that precedes a job in academia: teaching undergraduate mathematics, perhaps teaching graduate mathematics, maybe living for your research, belonging to a community of researchers in your field, and trips to conferences throughout the US and in Europe, South America, China and Japan (at someone else's expense, of course:-)

Is a Doctoral Degree required, or even a good idea, if you are interested in a job in Industry? There is no easy answer to this question. The doctoral degree can typically take anywhere from 3 years (fast) to 6 years (slow) past your Masters Degree. This means you are in school for anywhere from 5 to 8 years past your undergraduate degree. These are years you are not being paid at a full-time wage, and still following the academic routine. Whether you are willing to make this sacrifice is a very personal decision. However, the advantage of the Doctoral Degree when employed in Industry or Government is that you are considered at the top of hierarchy. For example, a Doctorate in Statistics opens the door to jobs with the US Government, where you are doing research for some government agency, such as for the FDA. These are very high-paying and secure work positions. Check out the list of employment for our doctoral graduates here: http://www.math.uic.edu/graduate/grad_people/alumni

Which Grad School?

Along with the question of whether to continue your studies in graduate school, there is the question of which grad school? There are many answers, but first, you must ask yourself what are your goals? Is the primary goal to stay in Chicago area? Then your choices are mainly limited to UIC, Northwestern University and the University of Chicago. Slightly further away is UIUC, Purdue and the University of Wisconsin.

The next consideration is which degree you are seeking, and in what subject. As a general rule, when pursuing a doctorate, you should hope to be admitted at one of the Group I Research Institutions, listed in the Appendices. The reasoning for recommending these schools, is that doing research in Mathematics is greatly facilitated if your *thesis supervisor* is a faculty member with a record of significant research. Such faculty are more prevalent at the Group I institutions. Also, Group I schools tend to have more active graduate programs, and offer better funding for their graduate students. Part of the graduate experience is provided by your fellow students, who are with you in your times of trial and triumph.

You can download a list of all the mathematics graduate programs in the US, which includes for each school what is required to apply for admission, and also lists useful information such as how many applicants they normally admit to their programs each year, how many Teaching Assistantships are available, and whether they offer Fellowship support for your studies. Plus, many universities take out advertisements in this pamphlet, to highlight the advantages of their graduate programs. The pamphlet is 28 MB in size, and is available from the AMS website here: http://www.ams.org/employment/asst.pdf

Given the large number of possible programs you might apply to, how do you choose? Not all group I schools are the same. For example, University of Chicago, Harvard University, Princeton University, Yale University, and UC Berkeley are considered the top five research Math programs. There is a hierarchy of all the 48 Group I schools, and this ranking *suggests* which are the "hottest" places to do math research. However, there are three major caveats:

- Will you get admitted? Will they offer you financial support?
- Do they offer an active research program in your field of interest? (e.g., number theory or fluid dynamics)
- What is it like to actually be a student in that city, or that department?

The best approach to getting information needed to answer these questions, is to check out the web sites for the departments you are seriously considering. Also, look at programs you are not considering, as this may raise questions or highlight concerns you had not thought of asking. In all cases, it is a personal decision, as you are going to live, study and work in this new city and environment for the next two to eight years.

The Web is one of the greatest boons towards choosing graduate programs in the last, well, forever. Here are three links to check out.

- The *Peterson's Guide to Graduate School* is a respected resource, with much helpful information. Check it out: http://www.petersons.com/graduate_home.asp

- The website *PhDs.org* offers advice on how to choose a graduate program and on how to decide whether to attend grad school. Check it out: http://www.phds.org/graduate-school/choosing-a-graduate-program/

- The Harvard Math Department offers a pamphlet *Graduate Schools and Fellowships in Mathematics* on choosing and applying to graduate school. They may be discussing Harvard at times, but the advice is universal. Check it out: http://abel.math.harvard.edu/pamphlets/gradsch.html

What To Do First?

This is easy – read the pamphlet Preparing for Graduate School prepared by Kari Dueball, Assistant Director of Graduate Studies in the MSCS Department at UIC. Dueball's advice is based on many years of handling graduate admissions, mentoring graduate students, and advising UIC undergraduates.

Next, you should sign up for the Graduate Record Exam (GRE) – both the General Exam, and most likely also the Math Subject Exam. Admission to most graduate programs in based in part on your GRE scores, some schools more than others. Some schools require the Subject Exam, others do not. However, an excellent score on the Subject Exam is always useful information to provide to the admissions committee, whether required or not. The results of your exams should be submitted by the application deadlines, or not long afterwards. These deadlines are typically anywhere from December 15 to January 15, for admission beginning the following August. The web site for the GRE is here: http://www.ets.org/gre/

The GRE Exams are given at several locations in the Chicago area, including on the UIC campus. Often available seats for the exams are limited, so sign up as soon as possible. For example, there is an exam given at UIC on November 8, 2008.

Admission to graduate programs is typically based on three factors: your courses and grades as an undergraduate; your GRE scores; and what can be the most important, letters of recommendation from faculty. This latter requirement may scare some or many undergraduates, especially if you are in classes where the Instructor for the class always "seems to be busy". Most Professors are always "busy", but many are happy to make time to discuss your graduate plans, and will do what they can to help you in the application process, including write a supporting letter.

First, there is a distinction to be made. At UIC, we have four types of course Instructors - Professors, Visiting Research Professors, Lecturers, and Graduate Students. As you take courses with higher level, your classes are more likely to be taught by a Professor, either tenured or visiting. By your senior year, you should have had courses from at least several Professors. The reason for pointing out this distinction, is that tenured faculty are more experienced at writing letters of recommendation, and hopefully can do the best job. Visiting faculty are often young researchers, pursuing their research interests in one of the fields of strength in our department. They also know how to write a letter of recommendation, although they may not have had the opportunity so often. Lecturers in our department may, or may not, be research mathematicians, and correspondingly may not be well-known in the math community. Finally, graduate students are still students, and are probably not the best resource for writing your recommendation letters. The point is that the admissions committee at the schools you apply for will take into consideration not just what your recommendation letters say, but also who wrote them.

Given all that, who do you ask to write your letters of recommendation? First, it should be someone who knows you, and your abilities as a student. If a faculty member writes a letter in support of you based just on your grade in some course, there is not much to say. Much better and easier for a faculty member, is to write a letter in support of a student who you have interacted with, whether it be about problems in some course, or questions you may have had about math, or just in general. So, get to know your Professors! Make an effort to talk with them. Ours is a wonderfully diverse faculty in the MSCS Department, with many research interests, and also widely diverse personal interests. Busy though faculty may seem, interrupt them to talk about your future career in Math!

The Math Club, Putnam Team and even the Friday Teas provide excellent opportunities for meeting faculty, so they know you are when it comes time to apply for graduate schools, as well as other jobs you may consider.

In any case, arrange for three letters of recommendation from faculty who you feel will say something positive in support of your application. Dueball's guidelines discusses the where-all of getting these letters, after you have decided whom to ask.

Who Pays? Can I get A Fellowship?

Attending graduate school in Mathematics has one great advantage over almost all other fields. Because so many other subjects require training in mathematics, from Calculus up through Ordinary Differential Equations and Linear Algebra, there is a great demand for Teaching Assistants to help with these courses. This means that when applying for graduate admission, you can also realistically hope to be offered a Teaching Assistantship which pays you a monthly stipend, along with your tuition.

In the MSCS Department at UIC, the stipend for a 50% TA position is $17,000 for nine months, with a possibility to obtain support for two summer months for many students. The stipend also includes a Tuition & Fee Waiver (TFW), so that you do not pay for graduate tuition, but do have to pay for some fees like health insurance. While this is not a great paying job, it is still livable. The rate of pay and duties for other graduate programs varies widely. The AMS online brochure [http://www.ams.org/employment/asst.pdf] lists this information for all graduate programs in Math in the US and Canada.

Many universities also offer Graduate Fellowships to their most outstanding applicants. These pay the same as a TA position, or sometimes much more ($30,000 for example) and include a TFW. These tend to be few in number, and if a graduate program you applied to offers you one, then they really want you!

There is one more option for funding - The National Science Foundation (NSF) Graduate Fellowship. These Fellowships offer 3 years of graduate support worth over $122,500. In addition, the university where you pursue your graduate studies with one of these Fellowships, traditionally pays your tuition also. It is a great deal! And as can be expected, these are highly competitive, and very prestigious. None-the-less, they are also distributed by region, so students applying from UIC have as good a chance of receiving one as from other schools.

The main requirements for submitting a competitive application is a very strong undergraduate record, and excellent letters of recommendation from hopefully three faculty members.

You can learn more about the NSF Graduate Fellowship at the main web page [http://www.nsfgrfp.org/].

If you are interested in applying for an NSF Graduate Fellowship, the Office of Undergraduate Studies in MSCS and the University Fellowship Office will be very enthusiastic in helping you prepare your application. The application deadline for mathematics is November 5, 2008, so it is time to get started with the application process now. Start by contacting the Director, Steve Hurder (dus@math.uic.edu) about applying, and we'll schedule a meeting quickly to discuss your next steps.

Group I Public

25 Departments - Ranked between 3.00 and 5.00 by the 1995 NRC Study
(List Last Updated January 2008)

Institution	Department
City University of New York, Graduate Center	Mathematics
Georgia Institute of Technology	Mathematics
Indiana University, Bloomington	Mathematics
Michigan State University	Mathematics
Ohio State University, Columbus	Mathematics
Pennsylvania State University	Mathematics
Purdue University	Mathematics
Rutgers University, New Brunswick	Mathematics
State University of New York, Stony Brook	Mathematics
University of California, Berkeley	Mathematics
University of California, Los Angeles	Mathematics
University of California, San Diego	Mathematics
University of California, Santa Barbara	Mathematics
University of Illinois, Chicago	Mathematics, Statistics & Computer Science
University of Illinois, Urbana-Champaign	Mathematics
University of Maryland, College Park	Mathematics
University of Michigan, Ann Arbor	Mathematics
University of Minnesota, Minneapolis	Mathematics
University of North Carolina, Chapel Hill	Mathematics
University of Oregon	Mathematics
University of Texas, Austin	Mathematics
University of Utah	Mathematics
University of Virginia	Mathematics
University of Washington	Mathematics
University of Wisconsin, Madison	Mathematics

Group I Private

23 Departments - Ranked Between 3.00 and 5.00 by the 1995 NRC Study
(List Last Updated January 2008)

Institution	Department
Boston University	Mathematics
Brandeis University	Mathematics
Brown University	Mathematics
California Institute of Technology	Mathematics
Carnegie Mellon University	Mathematical Sciences
Columbia University	Mathematics
Cornell University	Mathematics
Duke University	Mathematics
Harvard University	Mathematics
Johns Hopkins University, Baltimore	Mathematics
Massachusetts Institute of Technology	Mathematics
New York University, Courant Institute	Mathematical Sciences
Northwestern University	Mathematics
Princeton University	Mathematics
Rensselaer Polytechic Institute	Mathematical Sciences
Rice University	Mathematics
Stanford University	Mathematics
University of Chicago	Mathematics
University of Notre Dame	Mathematics
University of Pennsylvania	Mathematics
University of Southern California	Mathematics
Washington University	Mathematics
Yale University	Mathematics

Group IV

88 Departments (List Last Updated June 2008)

Institution	Department
Baylor University	Statistics
Boston University, School of Public Health	Biostatistics
Brown University	Statistical Sciences
Carnegie Mellon University	Statistics
Case Western Reserve University	Statistics
Case Western Reserve University	Epidemiology & Biostatistics
Colorado State University	Statistics
Columbia University	Statistics
Columbia University	Biostatistics
Cornell University	Biometrics
Duke University	Statistical Science
Emory University	Biostatistics
Florida State University	Statistics
George Washington University	Statistics
Harvard University	Statistics
Harvard University	Biostatistics
Iowa State University	Statistics
Johns Hopkins University	Biostatistics
Kansas State University	Statistics
Medical College of Wisconsin	Biostatistics
Medical University of South Carolina	Biometry & Epidemiology
Michigan State University	Statistics & Probability
New York University	Statistics & Operations Research, Stern School of Business
North Carolina State University	Statistics
North Dakota State University, Fargo	Statistics
Northwestern University	Statistics
Ohio State University, Columbus	Statistics
Oklahoma State University	Statistics
Oregon State University	Statistics
Pennsylvania State University, University Park	Statistics
Purdue University	Statistics
Rice University	Statistics
Rutgers University - New Brunswick	Statistics & Biotatistics
Southern Methodist University	Statistical Science
Stanford University	Statistics
State University of New York at Albany	Epidemiology & Biostatistics

SUNY at Buffalo	Biostatistics
Temple University	Statistics
Texas A&M University	Statistics
Tulane University	Biostatistics
University of Alabama at Birmingham	Biostatistics
University of Alabama - Tuscaloosa	Information Systems, Statistics, & Management Science
University of California, Berkeley	Statistics
University of California, Berkeley	Biostatistics
University of California, Davis	Statistics
University of California, Los Angeles	Statistics
University of California, Los Angeles	Biostatistics
University of California, Riverside	Statistics
University of California, Santa Barbara	Statistics & Applied Probability
University of Chicago	Statistics
University of Cincinnati	Epidemiology & Biostatistics, Medical College
University of Colorado at Denver	Preventive Medicine & Biometrics (Biostatistics Section)
University of Connecticut, Stoors	Statistics
University of Florida	Statistics
University of Georgia	Statistics
University of Illinois at Chicago	Epidemiology & Biostatistics
University of Illinois at Urbana-Champaign	Statistics
University of Iowa	Biostatistics
University of Iowa	Statistics & Actuarial Science
University of Kentucky	Statistics
University of Maryland, College Park	Measure Statistics & Evaluation
University of Massachusetts, Amherst	Biotatistics & Epidemiology
University of Michigan, Ann Arbor	Statistics
University of Michigan, Ann Arbor	Biostatistics
University of Minnesota - Twin Cities	Statistics
University of Minnesota - Twin Cities	Biostatistics
University of Missouri, Columbia	Statistics
University of Nebraska	Statistics
University of North Carolina at Chapel Hill	Statistics & Operations Research
University of North Carolina at Chapel Hill	Biostatistics
University of Oklahoma	Biostatistics & Epidemiology
University of Pennsylvania	Statistics
University of Pittsburgh	Statistics
University of Pittsburgh	Biostatistics
University of Rochester	Biostatistics
University of South Carolina	Statistics
University of South Carolina	Epidemiology & Biotatistics
University of Texas (Houston)	Biostatistics

University of Virginia	Statistics
University of Washington	Statistics
University of Washington	Biostatistics
University of Wisconsin, Madison	Statistics
University of Wyoming	Statistics
Virginia Commonwealth University	Biostatistics, Medical College
Virginia Polytechnic Institute & State University	Statistics
Western Michigan University	Statistics
Yale University	Statistics
Yale University	Biostatistics

A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas. --- G. K. Hardy (from *A Mathematician's Apology*, London 1941)

What Do Mathematicians Do?

Mathematicians are often asked by friends, family, colleagues in other fields, and strangers "What do mathematicians do?" Here are some resources and facts that may help answer that question.

"Some of What Mathematicians Do," by Martin H. Krieger, *Notices of the AMS*, November 2004, page 1226

"What Do Mathematicians Do?," by Professor A.J. Berrick, Department of Mathematics, National University of Singapore

Occupational Outlook Handbook: Mathematicians, U.S. Department of Labor, Bureau of Labor Statistics. The page includes significant points about the nature of the work.

Many people are familiar with mathematicians in academia, but mathematicians also work in many other fields, including:

- Astronomy and space exploration
- Climate study
- Medicine
- National security
- Robotics
- Animated films
- and in a wide range of businesses ("Theory Into Profit: Microsoft Invests in Mathematicians," by Allyn Jackson, *Notices of the AMS*, June/July 1998, and "Mathematical Experiences in Business, Industry and Government," by Phil Gustafson, *MAA Focus, March 2007*)

The diversity of fields that employ mathematicians is reflected in Mathematical Moments and Mathematics Awareness Month themes.

Mathematicians make it possible to send secure emails and buy things online. Mathematicians are essential to analyze data and design accurate models in fields as diverse as biology and finance. Mathematicians enabled researchers to complete The Human Genome Project quickly. And because of the prevalence of the computer at work and at play, mathematicians will continue to touch everyone in modern society.

How many mathematicians are there in the U.S.?

There are over 35,800 individual members of the four leading professional mathematical sciences societies in the U.S.--the AMS, the Association for Women in Mathematics, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics. Most would call themselves mathematicians, many received their doctoral degrees outside the U.S. There are at least 10,000 more members of the societies who are graduate students or in other categories, and there are also mathematicians who are not members of any of these societies.

Although they have advanced degrees in mathematics, many of those employed in academia might call themselves professors instead of mathematicians, and similarly, those in industry and government may not have "mathematician" in their job title. The job title doesn't tell the whole story, however--these people are doing mathematics and are indeed mathematicians. Furthermore, the number of mathematicians is increasing. The number of new Ph.D.'s in the U.S. has gone up every year since 2002.

Who *are* mathematicians?

Mathematicians are people of all ages and from all over the world who enjoy the challenge of a problem, who see the beauty in a pattern, a shape, a proof, a concept. Some of the best young mathematicians compete in math olympiads, state and national science fairs, or the fun *Who Wants to Be a Mathematician* game. Some high school mathematicians go to summer math camps to learn more and work with teams on projects; undergraduates participate in Summer Research Experiences. Many carry on their research and teach at colleges and universities, while others apply their skills in all kinds of professions (see Early Career Profiles, where former math majors discuss their careers, and Sloan Career Cornerstone, profiles of students just starting their career and those who have been working for decades). There's probably a bit of the "mathematician" in all of us and we don't even realize it. Keith Devlin poses this idea in a recent book, *The Math Instinct: Why You're a Mathematical Genius (along with Lobsters, Birds, Cats, and Dogs)*. In any case, those who are not "mathematicians" can appreciate the subject by reading reviews of books about mathematicians, "how mathematicians think," breakthroughs in mathematics, and current applications.

Resources on the mathematics profession:

- Data on the mathematics profession
- Employment services for mathematicians, including MathJobs and electronic job listings

What Do Mathematicians Do?

By Professor A J Berrick
Department of Mathematics
National University of Singapore

Note: *This article was first published in the October 1999 issue of the* Science Research Newsletter. *The newsletter is an online publication of the Faculty of Science, National University of Singapore.*

As a break from the tradition of this newsletter, this article is meant to provoke discussion. Most research mathematicians are quite passionate about their subject. Yet they are aware that their enthusiasm is not shared (to put it mildly) by the public at large, and even, in many cases, by research scientists. Is this just a case of "love is blind", or is it possible that mathematicians are aware of something about mathematics that outsiders are not? I'd like to investigate this matter in this article. I hope that in doing so I will stimulate others, both mathematicians and non-mathematicians, to think about these questions, and maybe even contribute their thoughts to later issues of the newsletter.

I think it's very important to start by asking the right question. Typically, academic disciplines are defined by their subject matter. So, to ask what a geologist does is more or less the same thing as to ask what a geologist studies. Thus, for the Oxford English Dictionary, it is "the science which has for its object the investigation of the earth's crust, of the strata which enter into its composition, with their mutual relations, and of the successive changes to which their present condition and positions are due". Similarly, for the OED, biochemistry is "the science dealing with the substances present in living organisms and with their relation to each other and to the life of the organism". Moving away from science, we have the OED's definition of architecture, "the art or science of building or constructing edifices of any kind for human use"; economics, the study of "the development and regulation of the material resources of a community or nation"; and linguistics, "the study of languages". These examples were chosen at random. In every case I expect that the reader's definition would be very similar to the one given by the dictionary.

Well, if such definitions are so easy for other disciplines, why not for mathematics? Like most people, the OED assumes that mathematics too can be defined by its subject matter and tries "the abstract science which investigates deductively the conclusions implicit in the elementary conceptions of spatial and numerical relations, and which includes as its main divisions geometry, arithmetic, and algebra; and, in a wider sense, so as to include those branches of physical or other research which consist in the application of this abstract science to concrete data". A good effort, but one gets the strong impression that whoever wrote it was struggling! The "spatial and numerical relations" obviously cover geometry and arithmetic, but then algebra had to be added because it wasn't dealt with. However, that's nowhere near good enough. Important "divisions" like analysis, probability, set theory and operational research are completely ignored by this definition, so clearly it's very inadequate. Should we compensate by listing the titles of, say, all mathematics modules taught at NUS, in the hope that we'll cover the subject that way? That attempt is doomed too, because a glance at the list soon reveals courses on topics like filter banks, chaos and fractals, cryptography, game theory, etc., that weren't there ten or twenty years ago. If the subject is to be defined by a list constructed at a certain time, then after that time, no newcomers can ever join. However, it's clear that mathematics is continuing to grow, its tentacles finding their way into areas of investigation previously thought beyond its reach.

The first attempt I heard to define mathematics by what it studies was by T G Room (a geometer commemorated by Room squares in combinatorics). He reckoned that mathematics is the study of relationships between concepts. Although this is helpful to the non-mathematician, it is clearly inaccurate. There are many concepts, like punishment and retribution, love and fidelity, whose relationships have failed to attract mathematical interest. In order to nail down those concepts that might yield mathematical investigation, the topologist D H Gottlieb claimed that mathematics is the study of well-defined things. This notion has some appeal to mathematicians, for whom the expression "well-defined" is part of the lingo, and for whom it excludes the above philosophical concepts. Yet I fear that an attempt to explain it to a non-mathematician would result in a "well-defined thing" as being "one that is amenable to mathematical inquiry". In other words, mathematicians study what mathematicians study.

Mathematicians tend to be pretty stubborn (we like to say that we persevere), but there comes a stage when one has unsuccessfully battled against a tough question for so long that one realises that the difficulty was simply that it was the wrong question in the first place. I believe that's what's happened here.

We shouldn't ask ***what*** a mathematician studies, we should ask ***how***.

Put it in another way; instead of the question, "What do mathematicians study?" we should ask, "What do mathematicians do?" Interestingly, when one examines the OED's attempt at a definition, one sees that, in contrast to the definitions of the other disciplines, there's a how answer only partly suppressed, "Investigates deductively the conclusions implicit in the elementary conceptions ...". Since mathematicians get their fingers into pies that often have names very different from geometry, arithmetic and algebra, it's more fruitful to clarify the process of doing mathematics. Then, when a new topic is proposed for the next revision of the mathematics curriculum, one has some hope of answering the question, "But is it mathematics?" My guess is that the correct answer would be, "Yes, when you look at it the right way." No geology lecture would be about zebras, or blood vessels, or language grammars, or DNA. By definition, it would fail to be a geology lecture. However, I've known mathematics lectures about these four topics.

Specifically, the lecture about **zebras** was interested in how they and other quadrupeds move. Different speeds of walking or running result in different sequences of hooves hitting the ground. Which sequences can occur, and what is the relation between the sequence and the speed?

Blood vessels can be studied for the way in which cells move along them; this is the dynamics of fluid motion where the walls are not rigid. And what governs the shape of the vessel itself? Can one predict when the forces will be so great as to lead to rupture?

Are there common rules of manipulation of words and phrases that apply across different **languages**? What does the similarity of such rules suggest about the cultural or genetic links between the speakers of such languages?

It's recently been discovered that in the process of replication, the enormously long **DNA** molecules get tied into knots, which partly dissolve and recombine as different knots. By inspecting the knots that appear, one can attempt deductions about the biochemical process that is leading from one knot to the next.

There's a pattern to what is happening in each of the above examples. The mathematician immediately ignores many specific features of the object in question. He or she is unlikely to care about whose body the blood vessel inhabits, or the age of the zebra. But pretty soon he or she may even forget that it's a blood vessel or zebra that's being studied, and may talk to a colleague about fluid in a tube or configurations of moving rods. The process of abstraction (OED: "of considering ... an attribute or quality independently of the substance to which it belongs") takes on a life of its own, so that before long two mathematicians may be discussing the problem in such a way that a third mathematician listening in would find it difficult to guess its physical origins. (The degree of difficulty is probably the distinction between pure and applied mathematics. Put like this, it's apparent that the distinction is more arbitrary and less clear-cut than generally recognised.)

After reading the above, the Japan-based mathematician A Kozlowski observed: "I think it a very important point that mathematics is probably the only subject whose content could change entirely and yet we would still recognize it as mathematics. We would probably recognize mathematics of beings from another universe, though we may have problems in distinguishing their physics from their philosophy, their history from their mythology etc."

I believe that the process of abstraction is a vital characteristic of mathematical thought, probably more distinctive than the method of deduction that the OED emphasises. Most scientists practise deduction, although not necessarily to the extent of mathematicians. However, other disciplines are comparatively restricted in the amount of abstraction that they allow themselves. While physiologists might be comfortable with the general properties of all blood vessels belonging to humans, or of all those with a certain condition, would they still be at ease with a level of abstraction that considered equally other fluids moving in inorganic tubes? Or would they feel that such generalizations were no longer in the realm of physiology?

In fact, the desire for abstraction seems to be an essential part of a mathematician's psyche. It's not just a matter of abstracting from the physical world to the mathematical; many mathematicians commence work only long after that process has been completed. Within mathematics, researchers are all the time striving to find just the right level of abstraction for a given setting, seeking the perfect balance between the twin goals of utility and generality.

Another feature of scientific method is of course induction, the attempt to generalize conclusions from a number of particular instances. Mathematicians practise this more often than is usually realised, however, in a special way. For a scientist such a conclusion has the status of a probationary law. If it stands the test of time, that is, accords with, and even predicts, subsequent observations, then it becomes more widely accepted. This tends to be a gradual process that can be partly or wholly reversed (medical science provides many examples of reversibility and controversy). For a mathematician, the result of induction is just a hunch. A strongly held hunch is honoured with the title of conjecture. For example, there was the Fermat Conjecture:

If x,y,z,n are whole numbers greater than 1 and $x^n + y^n = z^n$, then $n = 2$.

Evidently, Fermat produced this statement by induction after examining many specific cases (with no help from an electronic computer). (In the event that $n = 2$, then for any whole number k we can always take $x = 2k +1$, $y = 2k^2+2k$ and $z = y+1$.) After succeeding generations of mathematicians had played with it, the formulation received the status of a conjecture. For further generations it was widely believed to be a correct assertion, yet no mathematician would admit to the list of proved statements anything that was logically dependent on it. That all changed in this decade, when Andrew Wiles' famous proof (a chain of deductions occupying hundreds of printed pages) survived the rigorous checks of his peers. The statement is now called a theorem, and will always remain so. "Elevation to the theorage" is an irreversible process. The progression from hunch to conjecture is an example of scientific induction, but the final, irreversible graduation from conjecture to theorem has no parallel outside mathematics.

We have now distinguished three modes of thinking that highlight the difference between mathematics and other disciplines:

Abstraction,   Deduction,   Induction.

They are listed above in decreasing order of importance to mathematical research, but I would guess in increasing order of importance for scientists generally. One might object that if deduction and induction are seen as opposites, then why doesn't an opposite of abstraction appear in the list? Well, since abstraction consists in seeing common properties and patterns among different situations, then once one has obtained conclusions about the general, abstract setup, the process of deduction is enough to get one back to conclusions about the original, more concrete setting. So our list of three seems to be enough.

Just what the process of abstraction involves is a big topic, and not central to this article. At its heart seems to be one of the greatest joys of mathematical research - pattern recognition. The patterns are not usually the visual ones of everyday experience. Recall that humans also get excited by more subtle threads of similarity, for instance in hearing in a Wagnerian opera a musical motif that occurs elsewhere in the Ring Cycle. Mathematicians have available for painting their patterns the whole canvas of human experience. For example, the kinship ties of the Warlpiri people of the Australian outback exhibit the same pattern as the symmetries of a square (known algebraically as the dihedral group of order 8).

The drive to find common themes from disaparate areas seems to be part of the mathematician's temperament. At its most banal, it's a source of painful puns (like the one I had to resist earlier, after including the words "pie" and "fruitful" in the same sentence). Used more creatively, it helps to explain what has been called the "unreasonable effectiveness" of mathematics. Consider the following title of a research paper, by R Ghrist, that reached me today.

Configuration spaces ... it begins. I think: Yes, it's about topology.

... and ***braid groups*** ... Okay, about algebra too.

on ***graphs*** ... Combinatorics as well. This is getting pretty interesting. But now for the knockout blow ...

in ***robotics***.

The discovery of a unifying pattern can be like lightning flashing from one discipline to another. The difference is that it can illuminate both subjects forever. So there is a simple message for the nonmathematical researcher reading this article. When all seems cloudy, contact a mathematician (as broadminded as possible). Then stand by for flashes of lightning!

Prof A J Berrick
Department of Mathematics

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