MCS 590, Spring 2009

Homework 2

For a general nonlinear autonomous dynamical system

$$\dot{\vec{x}} = \vec{f}(\vec{x}) \tag{1}$$

the well-known 4-th order Runge-Kutta time integration method (RK4) with integration time step τ is given by the following formula:

$$\begin{aligned}
K_1 &= \vec{f}(\vec{x}_n), \\
K_2 &= \vec{f}(\vec{x}_n + \frac{\tau}{2}K_1), \\
K_3 &= \vec{f}(\vec{x}_n + \frac{\tau}{2}K_2), \\
K_4 &= \vec{f}(\vec{x}_n + \tau K_3), \\
\vec{x}_{n+1} &= \vec{x}_n + \frac{\tau}{6}(K_1 + 2K_2 + 2K_3 + K_4).
\end{aligned} \tag{2}$$

1. Derive the extended RK4 method, which computes the one-step tangent map $T(\vec{x}_n)$ in parallel with $\vec{x}_{n+1}$. Use the class notes, where I did the same thing for RK2.

2. Using initial condition $X_0 = 0.1, Y_0 = Z_0 = 0$ and the time step $\tau = 0.01$, for the Lorenz attractor with $\sigma = 10, r = 28, b = 8/3$ compute the n-step tangent maps $T^n_{\vec{x}_0}$ for $n = 50, 100, 500$ (which corresponds to the time $t = 0.5, 1, 5$). Compute singular values of these tangent maps.

3. Repeat the same steps for the Rössler attractor with $a = 0.2, b = 0.2, c = 5.7$ and the initial condition $X_0 = Y_0 = Z_0 = 0$.

4. Discuss the results.

Use octave/matlab or your favorite computer language for computations.