

Homework 9 solutions, Math 446, professor Agol, winter 2002

1.2.2.1 Show that if a disc is removed from the projective plane the result is a Mobius band. (In other words, a crosscap is a Mobius band.)

A projective plane is obtained by identifying two arcs in the boundary of a disk, so that the orientations of the edges agree. Removing a disc from a neighborhood of the vertex, we get a square with opposite sides glued with a twist, which is a Mobius band.

1.2.3.2 Show that the surface in figure 75 is also a Klein bottle.

Slitting the surface along the double points, we see an annulus, with the boundary circles identified by an orientation reversing identification, which is a Klein bottle.

1.2.3.3 Find a curve on the Klein bottle which separates it into two Mobius bands.

In figure 74, we see a Klein bottle given as a surface with two cross caps. Cutting along the circle separating the crosscaps, we get two punctured projective planes, which by 1.2.2.1 are Mobius bands.

What surface do you obtain by gluing the opposite edges of a polygon in a

1. orientation preserving fashion

When the polygon has $4n$ sides, one obtains a surface of genus n , with a cell structure with $2n$ edges and one vertex and one 2-cell. When the polygon has $4n + 2$ sides, there are two orbits of vertices under the gluing, and there are $2n + 1$ edges and 1 2-cell, so one has a genus n surface (by computing the Euler characteristic).

2. orientation reversing?

In this case, we are just gluing opposite points on the boundary of a disk, which gives a projective plane.