

Sample Midterm Math 535 Spring 2006

1. Find

$$\int_{|z|=4} \frac{z}{z^2 + 2z + 1} dz.$$

2. Show that if an analytic function $w = u + iv = f(z)$ takes all its values on the hyperbola $v = \frac{1}{u}$ then f is a constant

3. a)

Find the domain of convergence of the power series

$$\sum_{n=0}^{\infty} 2^{n/2} z^n.$$

- b) If $f(z) = \sum a_n z^n$ find a formula in terms of f for $\sum n^2 a_n z^{n+1}$.
4. If a polynomial $p(z)$ is real on the real axis, show that the roots of p come in conjugate pairs. (If z_0 is a root, then \bar{z}_0 is a root).
5. Find a domain Ω such that $\log(z)$ is an analytic function in Ω and $\int_{\gamma} \log(z) dz = 0$ for all closed curves $\gamma \subset \Omega$.