This week there were two talks at U. of C. on hyperbolic 3-manifolds. Juan Souto is visiting Jeff Brock and Chris Connell, so Peter Storm, a student of Dick Canary, came to visit as well. Pete will be at U. of Chicago next year on an NSF postdoc.

Juan Souto gave a talk on Thursday, on the Marden conjecture. My advisor, Mike Freedman, spent many years working on this conjecture. Marden didn't actually state it as a conjecture, but as a question [6]. It seems to be the last major step in obtaining a complete classification of Kleinian groups, with the solution of the ending lamination conjecture by Brock, Canary, and Minsky [7]. The Marden conjecture states that every hyperbolic 3-manifold with finitely generated fundamental group is topologically tame. By work of Bonahon, this is known in the case that the fundamental group is freely indecomposable. Souto proves this in the case that the manifold may be exhausted by compact cores, that is compact submanifolds which are homotopy equivalent to the manifold. Souto uses a trick of Canary, by taking a homologically trivial loop in the "Masur domain", and taking the two-fold branched cover to get a manifold with irreducible fundamental group and a negatively curved metric. Now, the lifts of boundaries of the compact cores disjoint from the branching loop become incompressible, so one may realize them as pleated surfaces. If the fundamental group of the original manifold was reducible, then the boundaries of the cores will contain loops which are homotopically trivial. Choosing a ruled negatively curved surface, such that the loop is totally geodesic, we see that the surface must intersect a compact part of the manifold. By a compactness argument for ruled negatively curved surfaces, the surfaces eventually become parallel, and therefore the cores downstairs are parallel proving tameness. Souto's theorem has a nice corollary that if one has a hyperbolic manifold which decomposes along a properly embedded incompressible surface of finite type into pieces whose covers are tame, then the manifold is tame as well. Souto pointed out in his talk that one may have non-tame manifolds that have a hyperbolic metric outside of a compact piece. To see this, choose a map $f : H \to H$ of a handlebody into itself, such that $H - int(f(H))$ is acylindrical, and $f_* : \pi_1(H) \to \pi_1(H)$ is the identity map. Then $H_\infty = \cup_{n\in\mathbb{N}\cup\{0\}} f^n(H)$ is a wild manifold homotopy equivalent to H. Then the manifold $L = H - int(f(H))/\{\partial H \sim \partial f(H)\}$ obtained by identifying by f restricted to ∂H has a hyperbolic metric, by Thurston's hyperbolization theorem. Then the infinite cyclic cover L_∞ of L dual to ∂H naturally decomposes as $L_\infty = \cup_{n\in\mathbb{Z}}(H - intf(H))_n$, where $\partial H \subset \partial(H - intf(H))_n$ is glued to $\partial f(H) \subset \partial(H - intf(H))_{n+1}$ by f. Thus, $H_\infty - H = \cup_{n\in\mathbb{N}}(H - intf(H))_n \subset L_\infty$ has a hyperbolic metric, and one may extend this metric over H to obtain a metric on H_∞ which is hyperbolic outside of a compact set. This example demonstrates the subtlety of the question.

The Marden conjecture has an important corollary for 3-manifold topologists who study compact 3-manifolds. If we take a cover of a compact hyperbolic 3-manifold with finitely generated fundamental group, then either the cover is geometrically finite, or it corresponds to a geometrically infinite surface subgroup which is the fiber of a finite sheeted cover of the manifold. This is a special case of Canary's covering theorem [3]. For example, this would imply that the figure 8 knot complement has LERF fundamental group, that is every finitely generated subgroup is the intersection of finite index subgroups containing it. It should have other implications, such as the presentation problem for finitely generated subgroups and the membership problem. I also have an application of Marden's conjecture to exceptional Dehn fillings which uses it both indirectly through the covering theorem, and directly, to apply paradoxical decomposition techniques of Culler and Shalen. Hopefully I'll discuss this at a later date.

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