Review of parts three and four of the course

In part three of the course we saw how to define functions and explored Maple's capabilities in differentiation and integration. We discussed Maple's data structures and solved equations in the fourth and final part on Maple.

Below is a list of questions to review. Consider also the review questions for the midterms, the quizzes, midterms, and homework assignments.

1. Use piecewise to define a function `int_inv_cub` which as function of the end points a and b always returns the correct value of $\int_a^b \frac{1}{x^3} dx$.
2. How can you make Maple return $5x^4 dx$ as a result of the command `diff(x^4,x)`?
3. The arc length of continuous function $f(x)$ over an interval $[a, b]$ can be defined as $\int_a^t \sqrt{1 + [f'(x)]^2}$.
   (a) Compute the arc length of the positive half of the unit circle, i.e.: $f(x) = \sqrt{1 - x^2}$ (answer $= \pi$).
   (b) Create a function (call it `arc_length`) in t which returns a 10-digit floating-point approximation of the arc length of the positive half of the circle, for $x \in [0, t]$.
4. Consider the recurrence relation
   $$h(n) = 5h(n - 1) - 6h(n - 2), \quad \text{for } n \geq 2, \quad \text{with } h(0) = 1 \text{ and } h(1) = -2.$$
   (a) The generating function $g(x) = \frac{1-7x}{1-5x+6x^2}$ defines $h(n)$ as the coefficient with x^n in the Taylor expansion of $g(x)$. Use $g(x)$ to define h as a function (call it `t`) of n which gives the value of $h(n)$.
   (b) Write a procedure to compute $h(n)$, directly using the recurrence relation from above. Make sure your procedure can compute $h(120)$. Compare with the result of (a).
   (c) Find an explicit expression for $h(n)$ as a function of n. Use this expression to define a function `s` which returns $h(n)$. Compare `s`(120) with `t`(120) and $h(120)$.
5. The Legendre polynomials are defined by
   $$P_0(x) = 1, \quad P_1(x) = x, \quad P_n(x) = \frac{2n - 1}{n} x P_{n-1}(x) - \frac{n - 1}{n} P_{n-2}(x), \text{ for } n \geq 2.$$
   Write a efficient recursive procedure `legendre` to compute $P_n(x)$. The procedure `legendre` takes on input the variable x and as index the degree n of P_n.
   Compare the output of `legendre`[50](x) with `orthopoly[P](50,x)`.
6. What is the main difference between automatic and symbolic differentiation? In which circumstances do we prefer the result of an automatic differentiation? Give a good example to illustrate your point.
7. The command **stats[random,uniform[-1,1]]** allows us to generate random numbers, uniformly distributed in $[-1, 1]$.
   (a) Create two lists, call them `lx` and `ly`, each with 100 hundred random points, uniformly sampled from $[-1, 1]$.
   (b) Join the two lists `lx` and `ly` into one list of points. Each point is represented by a list of two coordinates, one from the list `lx`, and the other from the list `ly`.
   (c) Select from the list of points all points inside the unit circle $x^2 + y^2 = 1$.
   How many such points do you find?
8. Suppose we want to plot the curve $x^4 + x^2y^2 - y^2 = 0$ for x and y both between -1 and $+1$.

(a) Sampling this curve as given in rectangular coordinates, how many samples do we need to take from the curve to obtain a nice plot?

(b) Convert the curve into polar coordinates and plot. Give all commands used to obtain the plot. How many samples of the curve are needed here?

9. Solve $x^2a^2 - 2x^2a - 3x^2 - xa^2 + 4xa - 3x + a^2 + 2a - 15$ for x for all values of the parameter a. Be as complete as possible in your description of the solution.

10. Find the point with real coordinates on the curve $xy - 2x + 3$ which lies closest to the origin.

11. How many real solutions does the system $\begin{cases} x^2 - 2y^2 - 1 = 0 \\ xy - 2x - 3 = 0 \end{cases}$ have?

12. Consider $y'' + 6y' + 13y = 0$, with $y(\pi/2) = -2$ and $y'(\pi/2) = 8$.

(a) Find an exact solution to this initial value problem and use this to create a function `s` which returns a numerical 10-digit floating-point approximation of the solution.

(b) Solve this initial value problem numerically. Compare the solution with the value for $y(2)$ and also with `s(2)` obtained in (a).

13. Create a 5-by-5 Vandermonde matrix where the (i, j)-th entry is x_i^{j-1}. Show that its determinant equals $(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)(x_1 - x_5)(x_2 - x_3)(x_2 - x_4)(x_2 - x_5)(x_3 - x_4)(x_3 - x_5)(x_4 - x_5)$.

FINAL EXAM is on Tuesday, May 5, 2009, 10:30 AM - 12:30 PM