October 31 worksheet

(1) Rewrite the following as a single double integral:

$$\int_1^2 \int_{-\frac{1}{\pi}\cos^{-1}(\frac{y}{2})}^{\frac{1}{\pi}\cos^{-1}(\frac{y}{2})} f(x,y)dx\,dy$$

$$+\int_0^1 \left(\int_{-\frac{1}{\pi}\cos^{-1}(\frac{y}{2})}^{-\sqrt{1-y}} f(x,y)dx + \int_{\sqrt{1-y}}^{\frac{1}{\pi}\cos^{-1}(\frac{y}{2})} f(x,y)dx \right) dy.$$

Answer: The key to this exercise is recognizing what the region is over which the function f is being integrated. The answer is:

$$\int_{-1}^{1} \int_{1-x^2}^{2\cos(\frac{\pi x}{2})} f(x,y)dy\,dx.$$

(2) Set up the iterated integrals in both orders to integrate $f(x,y)$ over the given regions.

(a) The region bounded by the lines $x = 0$, $y = 0$ and $y = mx+b$ where $m > 0$ and $b > 0$.
Answer: One answer is:

$$\int_0^{-b/m} \int_0^{mx+b} f(x,y)dydx.$$

(b) The region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
Answer:

$$\int_{-a}^{a} \int_{-b\sqrt{1-\frac{x^2}{a^2}}}^{-b\sqrt{1-\frac{x^2}{a^2}}} f(x,y)dydx.$$

(c) The region inside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and outside the ellipse $\frac{x^2}{c^2} + \frac{y^2}{d^2} = 1$ where $c < a$ and $d < b$.
Answer: You need at least four terms to do this one in rectangular coordinates. Here's one possible answer:

$$\int_{-b}^{-d} \int_{-a\sqrt{1-\frac{y^2}{b^2}}}^{a\sqrt{1-\frac{y^2}{b^2}}} f(x,y)dxdy$$

$$+\int_{-d}^{d} \left(\int_{-a\sqrt{1-\frac{y^2}{b^2}}}^{-c\sqrt{1-\frac{y^2}{d^2}}} f(x,y)dx + \int_{c\sqrt{1-\frac{y^2}{d^2}}}^{a\sqrt{1-\frac{y^2}{b^2}}} f(x,y)dx \right) dy$$

$$+\int_{d}^{b} \int_{-a\sqrt{1-\frac{y^2}{b^2}}}^{a\sqrt{1-\frac{y^2}{b^2}}} f(x,y)dxdy$$

(3) Set up the triple iterated integrals to integrate the function $g(x, y, z)$ over the described regions.

(a) The tetrahedron formed by the coordinate planes and the plane $ax+by+cz+d=0$ where a, b, c, and d are all greater than 0.

Answer: In class, we were all drawing the wrong picture.

$$\int_{-d/c}^{0}\int_{\frac{-d-cz}{b}}^{0}\int_{\frac{-by-cz-d}{a}}^{0} g(x, y, z)dxdydz.$$

(b) The region bounded by the xz plane and the surface $y = 1-(\frac{x^2}{a^2}+\frac{y^2}{b^2})$.

Answer Here's one possible answer:

$$\int_{-b}^{b}\int_{-a\sqrt{1-\frac{z^2}{b^2}}}^{a\sqrt{1-\frac{z^2}{b^2}}}\int_{0}^{1-(\frac{x^2}{a^2}+\frac{y^2}{b^2})} g(x, y, z)dzdxdy.$$

(c) The region above the xy plane, below the cone $z^2 = x^2+y^2$ and inside the cylinder $\sqrt{x^2+y^2} = a^2$.

Non-answer: No one attempted this one in class, so for now at least, the answer will be withheld...