Discussion problems for November 16

(1) Evaluate $\int_C (2x+9z)\,ds$ where C is given by $\mathbf{r}(t) = t\mathbf{i}+t^2\mathbf{j}+t^3\mathbf{k}$, $0 \le t \le 1$.

Solution: First find $|\mathbf{r}'(t)$:

$$|\mathbf{r}'(t)| = | < 1, 2t, 3t^2 > | = \sqrt{1 + 4t^2 + 9t^4}.$$

Now we can compute the integral:

$$\int_C (2x + 9z)\,ds = \int_0^1 (2t + 9t^3)\sqrt{1 + 4t^2 + 9t^4}\,dt = \frac{14\sqrt{14} - 1}{6},$$

where the integration is done by substitution.

(2) Find the mass and center of mass of a wire in the shape of $x^2 + y^2 = r^2$ for $x \ge 0$, $y \ge 0$ if the density is $\rho(x, y) = x + y$.

Solution: The mass is given by calculating the line integral of the density over the wire. Parameterize the wire by $\mathbf{r}(t) =< r\cos t, r\sin t >$. Then $m = \int_0^{\pi/2} r^2(\cos t + \sin t)\,dt = 2r^2$.

The center of mass has x and y coordinates equal by symmetry. Its coordinates are $(\bar{x}/m, \bar{x}/m)$ where $\bar{x} = \int_C x(x + y)\,ds$. Calculating, you should find that the center of mass has coordinates $(\frac{r(\pi+2)}{8}, \frac{r(\pi+2)}{8})$.