

**Solution to 14.4, number 22:** The problem is to show that the velocity and acceleration vectors of a particle moving at constant speed are orthogonal. Suppose that the vector valued function  $\mathbf{r}(\mathbf{t})$  gives the position vector of the particle at time  $t$ . Then the velocity vector at time  $t$  is  $\mathbf{v}(\mathbf{t}) = \mathbf{r}'(\mathbf{t})$ . Also,  $\mathbf{a}(\mathbf{t}) = \mathbf{v}'(\mathbf{t})$ , so

$$\mathbf{v}(\mathbf{t}) \cdot \mathbf{a}(\mathbf{t}) = \mathbf{v}(\mathbf{t}) \cdot \mathbf{v}'(\mathbf{t}).$$

Now using the constant speed assumption, we know that the length of  $\mathbf{v}(\mathbf{t})$  is constant. Therefore, by example 5 in section 14.2 which states that the dot product of a vector-valued function of constant length with its derivative is zero, we see that

$$\mathbf{v}(\mathbf{t}) \cdot \mathbf{v}'(\mathbf{t}) = \mathbf{0},$$

which implies that the velocity and acceleration vectors are orthogonal.