2005 Solutions

1. (a) The magnitude of the cross product of the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ gives the area of the parallelogram spanned by them, so the area of the triangle is half this quantity: $\sqrt{294}/2$

   (b) The dot product of the vector $\overrightarrow{BC}$ with the vector $\overrightarrow{AB}$ is 0, so the form a right angle, hence the triangle is right.

2. The line contains the vector $< 4, 2, 1 >$ (see the definition of the symmetric equations for a line). The line also contains the point $(-1, 4, 1)$. This point and the other point which is said to be in the plane form the vector $< 3, -5, 4 >$ Then the cross product of $< 4, 2, 1 >$ and $< 3, -5, 4 >$ gives a normal vector to the plane, $\mathbf{n} =< 13, -13, -26 >$. Therefore the plane can be given by either of the following equations:

$$13(x - 2) - 13(y + 1) - 26(z - 5) = 0$$

$$13(x + 1) - 13(y - 4) - 26(z - 1) = 0.$$

3. $\mathbf{v}(t) =< 1, 2t, 3t^2 >$, $\nu(t) = \sqrt{1 + (2t)^2 + (3t^2)^2}$, and $\mathbf{a}(t) =< 0, 2, 6t >$.

4. Set $f(x, y)$ equal to k and manipulate algebraically to get

$$x^2 + y^2 = \frac{k - 2}{4}.$$

   Then when $k = 2$ the level curve is a circle of radius 0, or just the origin, when $k = 4$ the level curve is a circle of radius $\sqrt{2}/2$ about the origin, and when $k = 10$, the level curve is a circle of radius $\sqrt{2}$ about the origin.

5. This limit does not exist. Check that approaching $(0, 0)$ along the line $y = 0$ gives 0, while approaching $(0, 0)$ along the line $y = x$ gives $1/2$.

6. $f_y = -e^{2x} \sin(y)$, $f_{xy} = -2e^{2x} \sin(y)$, and $f_{yy} = -e^{2x} \cos(y)$.

7. The curvature is greatest at B the unit tangent vector is turning fastest per arclength which implies that $|dT/ds|$ is greatest, where s is arclength.