Here's how you can prove the triangle inequality, $|\mathbf{a}+\mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$. First, let's look at the square of the left side:

$$
\begin{aligned}
|\mathbf{a}+\mathbf{b}|^2 &= (\mathbf{a}+\mathbf{b}) \cdot (\mathbf{a}+\mathbf{b}) \\
&= \mathbf{a} \cdot \mathbf{a} + 2(\mathbf{a} \cdot \mathbf{b}) + \mathbf{b} \cdot \mathbf{b} \\
&= |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2(\mathbf{a} \cdot \mathbf{b}).
\end{aligned}
$$

Now, look at the square of the right hand side:

$$(|\mathbf{a}| + |\mathbf{b}|)^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2|\mathbf{a}||\mathbf{b}|.$$

Comparing this to the left side, we see that the first two terms are identical, and the third term of the left hand side is less than the third term of the right hand side by the Cauch-Schwartz inequality, which everyone was able to figure out.