Math 210 Summer 1992 — Calculus III — Text: J. Stewart, Calculus (1991)

SIGNATURE = _____. *Soc. Sec. No.* _____.

INSTRUCTIONS:

1. The examination consists of 12 questions and contains 8 pages, not including this cover sheet. Check your examination paper to see if it contains 8 pages. The exam is worth 200 points or about 25 points per page. **PRINT** your name on every page.

2. Show all work, and put your answer in the space provided.

3. Write your name on each page of the examination paper. Do your work in the space provided. If more space is needed, use the back of the preceding page and mark the space to indicate this.

4. *Keep your eyes on your own paper and keep your own paper covered. Receiving or giving aid in a final examination is a cause for dismissal from the University.*

FOR GRADING PURPOSES ONLY

problem	1	2	3	4	5	6	7	8	9	10	11	12
score												

TOTAL:

1. Suppose $\vec{u} =< 2, -3, 4 >$ and $\vec{v} =< 4, 5, 6 >$.

   (a) Find the Cosine of the Angle between $\vec{u}$ and $\vec{v}$;

1(a). answer:

   (b) Find the Length of the Projection of $\vec{u}$ on $\vec{v}$;

1(b). answer:

2. Let $P = (1, 0, 0)$, $Q = (0, 2, 0)$, and $R = (0, 0, 1)$.

   (a) Find the Equation of the Plane passing through the Points P, Q and R.

2(a). answer:

   (b) Find the Distance from the Origin to the Plane in part (b).

2(b). answer:

3. Given the Position Vector $\vec{r}(t) = \sin(2t)\vec{i} - \cos(2t)\vec{j} + 4t\vec{k}$, find the

(a) Velocity =

3(a). answer:

(b) Acceleration =

3(b). answer:

(c) Speed =

3(c). answer:

(d) Unit Tangent Vector =

3(d). answer:

(e) Arclength along the path from $t = 0$ to $t = \pi$.

3(e). answer:

(f) Curvature =

3(f). answer:

(g) Unit Normal Vector =

3(g). answer:

4. For the Function $f(x, y, z) = 2\sin(xyz) + 2z^2/x$, find

(a) $\dfrac{\partial f}{\partial z} =$

4(a). answer:

(b) $\dfrac{\partial f}{\partial y} =$

4(b). answer:

5. Find the Equation of the Tangent Plane to the Surface defined by

$$F(x, y, z) = x^3 + 2xy - z^3 - 2zx + 28 = 0$$

at the Point $P = (1, 2, 3)$.

5. answer:

6. Suppose that

$$T(x, y, z) = \exp(-x^2 - y^3 - z^4)$$

represents the Temperature at the point with coordinates (x, y, z). Suppose that you are standing at the Point P with coordinates $(-1, -1, -1)$.

(a) In which Direction is the Temperature Increasing most rapidly?

6(a). answer:

(b) In which Direction is the Temperature Decreasing most rapidly?

6(b). answer:

(c) How Fast is the Temperature Changing in the direction of the Vector $\vec{v} =< 3, 4, 5 >$?

6(c). answer:

7. Locate and Classify *All* Critical Points of the function

$$f(x, y) = x^3 - 3xy + y^3.$$

7(a). Locations:

7(b). Classifications:

8. Evaluate the following Integral: (hint: reverse the order of integration.)

$$\int_{y=0}^{y=2} \int_{x=y}^{x=2} \sin(-x^2) dx dy =$$

8. answer:

9. Evaluate the following Integral (hint: think polar):

$$\int_{y=-1}^{y=1} \int_{x=0}^{x=\sqrt{1-y^2}} \sin(x^2+y^2)\ dx dy =$$

9. answer:

10. Find the Volume of the Solid that lies above the Cone $\phi = \pi/3$ and below the Sphere $\rho = 4\cos\phi$.

10. answer:

11. Give 5 other iterated integrals that are equal to the iterated integral

$$\int_{y=0}^{y=1} \int_{x=y}^{x=1} \int_{z=0}^{z=y} f(x,y,z)\, dz\, dx\, dy.$$

12. Evaluate

$$\int\int\int x^2 z y^2 dV$$

over that part of the Cylinder $x^2 + z^2 = 9$ in the First Octant to the left of the plane $y = 2$.

12. answer: