Math 180: Calculus I **Fall 2014**

October 16

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1. Sketch a function that is continuous on $(-\infty, \infty)$ with the following conditions: $f'(-1)$ is undefined; $f'(x) > 0$ on $(-\infty, -1)$; $f'(x) < 0$ on $(-1, \infty)$.

2. On what intervals is $f(x)$ increasing / decreasing? On what intervals is the function concave up / concave down? Identify all critical points and inflection points. Use the second derivative test to classify critical points if possible.

   (a) $f(x) = x^4 - 4x^3$

   (b) $f(x) = \cos^2 x$ on $[-\pi, \pi]$

   (c) $f(x) = x^2 - 2\ln x$

3. Explain why the following statements are true, or provide a counterexample.

   (a) If $f''(a) = 0$, then f has an inflection point at a.

   (b) If $f(x) = g(x) + c$ for some constant c, then f and g increase and decrease on the same intervals.

   (c) If f and g both increase on an interval, then the product fg also increases on that interval.

4. Can a continuous function on $(-\infty, \infty)$ have exactly four zeros and two local extrema?

5. For a general parabola $f(x) = ax^2 + bx + c$, for what values of a, b and c is the parabola concave up, and for what values is it concave down?