Math 180: Calculus I **Fall 2014**

November 18

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1. Express as a definite integral:
   $\lim_{\Delta\to 0}\sum_{k=1}^{n}(x_k^{*2}+1)\Delta x_k$ on $[0,2]$
   SOLUTION:
   $$\int_0^2 (x^2+1)dx$$

2. Suppose $\int_1^4 f(x)dx = 8$ and $\int_1^6 f(x)dx = 5$. Evaluate the following integrals
   (a) $\int_1^4(-3f(x))dx$
   SOLUTION:
   $$\begin{aligned}\int_1^4(-3f(x))dx &= -3\int_1^4 f(x)dx\\ &= -3\cdot 8\\ &= -24\end{aligned}$$
   (b) $\int_6^4 12f(x)dx$
   SOLUTION:
   $$\begin{aligned}\int_6^4 12f(x)dx &= -\int_4^6 12f(x)dx\\ &= -12\int_4^6 f(x)dx\\ &= -12\left(\int_1^6 f(x)dx - \int_1^4 f(x)dx\right)\\ &= -12(5-8)\\ &= 36\end{aligned}$$

3. Consider two functions f and g on $[1,6]$ such that
   $\int_1^6 f(x)dx = 10, \int_1^6 g(x)dx = 5,$
   $\int_4^6 f(x)dx = 5$, and $\int_1^4 g(x)dx = 2$.
   Evaluate the following
   (a) $\int_1^4(f(x)-g(x))dx$
   SOLUTION:
   $$\begin{aligned}\int_1^4(f(x)-g(x))dx &= \int_1^4 f(x)dx - \int_1^4 g(x)dx\\ &= \left(\int_1^6 f(x)dx - \int_4^6 f(x)dx\right) - 2\\ &= (10-5)-2\\ &= 3\end{aligned}$$

(b) $\int_4^6 (g(x) - f(x))dx$
SOLUTION:

$$\begin{aligned}\int_4^6 (g(x) - f(x))dx &= \int_4^6 g(x)dx - \int_4^6 f(x)dx \\ &= \left(\int_1^6 g(x)dx - \int_1^4 g(x)dx\right) - 5 \\ &= (5-2) - 5 \\ &= -2\end{aligned}$$

(c) $\int_4^1 2f(x)dx$
SOLUTION:

$$\begin{aligned}\int_4^1 2f(x)dx &= -\int_1^4 2f(x)dx \\ &= -2\int_1^4 f(x)dx \\ &= -2\left(\int_1^6 f(x)dx - \int_4^6 f(x)dx\right) \\ &= -2(10-5) \\ &= -10\end{aligned}$$

4. Use the definition of a definite integral as a limit of a left Riemann sum to evaluate the definite integrals

(a) $\int_1^3 (2x+1)dx$
SOLUTION: Suppose the width of each subinterval is Δ. Then there will be $(3-1)/\Delta = 2/\Delta$ intervals. The right endpoint of the kth interval will be $1 + k \cdot \Delta$, and the rectangle's height will be $f(1 + k\Delta) = 2(1 + k\Delta) + 1 = 2k\Delta + 3$. We can now set up the summation.

$$\begin{aligned}\int_1^3 (2x+1)dx &= \lim_{\Delta\to 0} \sum_{k=1}^{2/\Delta} (2k\Delta + 3)\Delta \\ &= \lim_{\Delta\to 0} \sum_{k=1}^{2/\Delta} (2k\Delta^2 + 3\Delta) \\ &= \lim_{\Delta\to 0} \left(2\Delta^2 \sum_{k=1}^{2/\Delta} k + \Delta \sum_{k=1}^{2/\Delta} 3\right) \\ &= \lim_{\Delta\to 0} \left(2\Delta^2 \frac{\frac{2}{\Delta}\frac{2+\Delta}{\Delta}}{2} + \Delta\frac{2}{\Delta}3\right) \\ &= \lim_{\Delta\to 0} (4 + 2\Delta + 6) \\ &= 10\end{aligned}$$

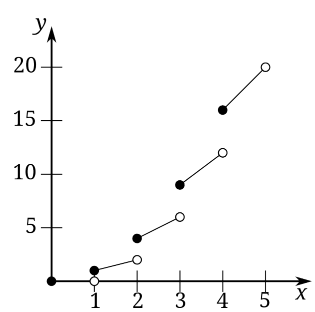
(b) $\int_0^2 (x^2 - 1)dx$
SOLUTION: If the width of each subinterval is Δ, then there will be $2/\Delta$ intervals. The right endpoint of the kth interval is $k\Delta$ and the height of the rectangle will be $f(k\Delta) = k^2\Delta^2 - 1$. We

can set up the summation as follows:

$$\begin{aligned}\int_0^2 (x^2-1)dx &= \lim_{\Delta\to 0}\sum_{k=1}^{2/\Delta}(k^2\Delta^2-1)\Delta \\ &= \lim_{\Delta\to 0}\sum_{k=1}^{2/\Delta}\left(k^2\Delta^3-\Delta\right) \\ &= \lim_{\Delta\to 0}\left(\Delta^3\sum_{k=1}^{2/\Delta}k^2-\Delta\sum_{k=1}^{2/\Delta}1\right) \\ &= \lim_{\Delta\to 0}\left(\Delta^3\frac{\frac{2}{\Delta}\frac{2+\Delta}{\Delta}\frac{4+\Delta}{\Delta}}{6}-\Delta\frac{2}{\Delta}\right) \\ &= \lim_{\Delta\to 0}\left(\frac{16+12\Delta+2\Delta^2}{6}-2\right) \\ &= \frac{8}{3}-2 \\ &= \frac{2}{3}\end{aligned}$$

5. Remember the floor function $\lfloor x\rfloor$ is the greatest integer less than or equal to x. Evaluate the following integral:

$$\int_1^5 x\lfloor x\rfloor dx$$

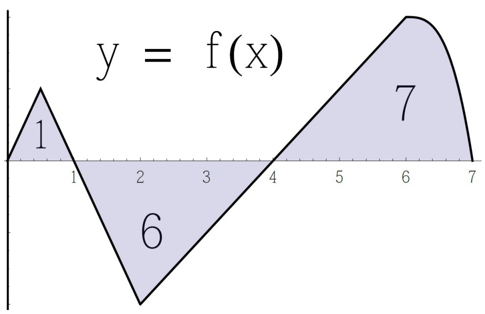


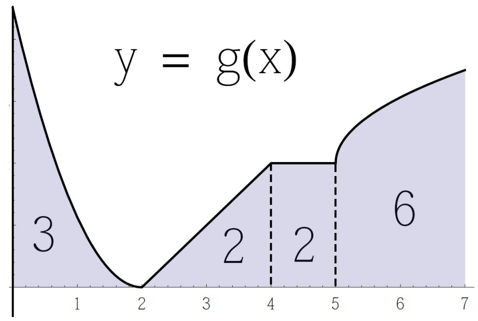
6. The definite integral can be computed as the area of the 5 trapezoids. The area of a trapezoid is $b\frac{h_1+h_2}{2}$, that is the base times the average of the two heights. Since the bases of each measures 1, then the average of the heights is the area. The heights can be calculated as follows:

rectangle	x_1	slope	h_1	h_2	$\frac{h_1+h_2}{2}$
1	0	0	0	0	0
2	1	1	1	2	1.5
3	2	2	4	6	5
4	3	3	9	12	10.5
5	4	4	16	20	18

So the total area is $1.5+5+10.5+18=35$.

7. The graphs below represent two functions, $f(x)$ and $g(x)$ and the values inside the enclosed portions represent the areas of those portions.





(a) Compute $\int_0^4 (2f(x) + 3g(x))dx$
SOLUTION:

$$\begin{aligned}\int_0^4 (2f(x) + 3g(x))dx &= \int_0^4 f(x)dx + \int_0^4 3g(x)dx \\ &= 2\int_0^4 f(x)dx + 3\int_0^4 g(x) \\ &= 2(1-6) + 3(3+2) \\ &= -10 + 15 \\ &= 5\end{aligned}$$

(b) Compute $\int_7^4 f(x)dx$

$$\begin{aligned}\int_7^4 f(x)dx &= -\int_4^7 f(x)dx \\ &= -7\end{aligned}$$