Math 180: Calculus I **Fall 2014**

September 23

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1. Evaluate the following limits using the facts that:

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \quad \text{and} \quad \lim_{x\to 0}\frac{\cos x-1}{x}=0$$

   (a) $\lim_{x\to 0}\frac{\tan 5x}{x}$

   (b) $\lim_{x\to -3}\frac{\sin(x+3)}{x^2+8x+15}$

2. Evaluate the derivatives dy/dx

   (a) $y=\frac{\cos x}{\sin x+1}$

   (b) $y=\csc x$ using the quotient rule

   (c) $y=\frac{\cot x}{1+\csc x}$

   (d) $y=\frac{x\cos x}{1+x^3}$

3. Evaluate the following limit or state it does not exist.

$$\lim_{x\to 0}\frac{\sin ax}{\sin bx}, \text{ where } a \text{ and } b \text{ are constants with } b\neq 0$$

4. Find the following derivatives using the product rule

$$\frac{d}{dx}(\sin^2 x) \quad \frac{d}{dx}(\sin^3 x) \quad \frac{d}{dx}(\sin^4 x)$$

   Make a conjecture about $\frac{d}{dx}(\sin^n x)$. See if you can prove it by induction!

5. Use the fact that $\cos(x+h)=\cos(x)\cos(h)-\sin(x)\sin(h)$ to prove that $\frac{d}{dx}\cos x=-\sin x$ using the limit definition.