

September 30

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1. Find the derivative of the following using the chain rule.

(a) $y = \sin^5 x$

(b) $y = \tan(5x^2)$

(c) $y = \sin(4 \cos x)$

(d) $y = (\sec x + \tan x)^4$

2. Consider the table

x	1	2	3	4	5
$f'(x)$	-6	-3	8	7	2
$g(x)$	4	1	5	2	3
$g'(x)$	9	7	3	-1	-5

Let $h(x) = f(g(x))$, and $k(x) = g(g(x))$. Compute the following derivatives:

(a) $h'(1)$

(b) $k'(5)$

3. Find the derivative using repeated applications of the chain rule:

(a) $y = \sin(\sin(e^x))$

(b) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

(c) $y = \left(\frac{x}{x+1}\right)^5$

4. Find the second derivative:

(a) $y = x \cos(x^2)$

(b) $y = \sqrt{x^2 + 2}$

5. $y''(t) + 2y'(t) + 5y(t) = 0$ is a differential equation. Verify that a solution to the differential equation is

$$y(t) = e^{-t} (\sin(2t) - 2 \cos(2t)).$$

6. Derive a formula for $\frac{d^2}{dx^2} f(g(x))$ using the chain rule and product rule, and use this formula to calculate

$$\frac{d^2}{dx^2} \sin(x^4 + 5x^2 + 2).$$