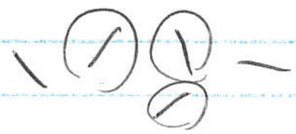
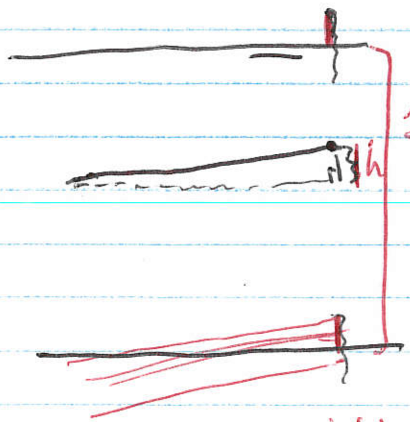
Stat 381 Feb 27

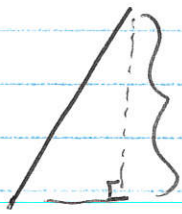
Buffon's Needles



needles length 1"
lines 1" apart

What is the prob a needle crosses a line.

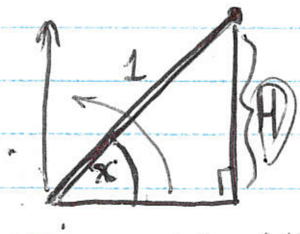




Prob a needle crosses is $\frac{|h|}{1}$

$0 \le |h| \le 1$

expected height.



$X \sim Unif_c(0, \frac{\pi}{2})$

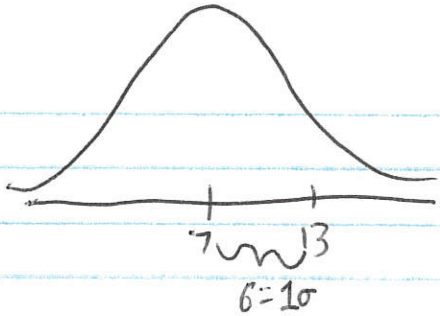
$0 \le x \le \frac{\pi}{2}$ $\quad f(x) = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi} \approx .6366$

$E(H)$ $\quad H = \sin x$

$= E(\sin X) = \int_0^{\frac{\pi}{2}} (\sin X) \frac{2}{\pi} dx = \frac{2}{\pi}\left[-\cos x\right]_0^{\frac{\pi}{2}}$

$= \frac{2}{\pi}[0 - -1] = \frac{2}{\pi}$

$X \sim N(\mu = 7, \sigma^2 = 36)$



$P(X > 13)$?

To Standardize X

how many std. dev's from the mean.

$$z = \frac{x - \mu}{\sigma}$$

Standardize 13

$$z = \frac{13 - 7}{\sqrt{36}} = \frac{13-7}{6} = 1$$

$\sigma^2 = 36$

$\sigma = \sqrt{36} = 6$

<u>Thm</u> For $X \sim N(\mu, \sigma^2)$

$$P(X \le x) = P\left(Z \le \overset{z}{\downarrow}\frac{x-\mu}{\sigma}\right)$$

proof

$$P(X \le x_1) = \int_{-\infty}^{x_1} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx$$

use $z = \frac{x-\mu}{\sigma} = \frac{1}{\sigma}x - \frac{\mu}{\sigma}$

so $dz = \frac{dx}{\sigma}$

$$= \int_{-\infty}^{z_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

$z^2 = \frac{(x-\mu)^2}{\sigma^2}$

$$= P(Z \le z_1)$$

$P(X \le x) = P\left(Z \le z = \frac{x-\mu}{\sigma}\right)$

$P(X \le 13) = P(Z \le 1)$ $\quad 1 = \frac{13-7}{6}$

$P(X > 13) = 1 - P(X \le 13) = 1 - P(Z \le 1)$

$= 1 - (.8413)$

Lower upper

$= 1 - \text{normalcdf}(-10, 1)$

TI 84/83 $\text{normalcdf}(a, b, \mu, \sigma)$

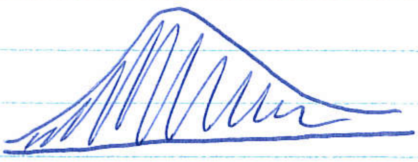
$= P(a \le X \le b)$ for $X \sim N(\mu, \sigma^2)$

∞

$P(X > 13) = \text{normalcdf}(13, 10000, 7, 6)$

$13 + 10 \cdot \sigma$ $\quad$ σ not σ^2

$P(Z > 2)$

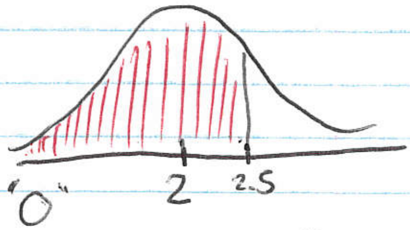
$P(-\infty < Z < \infty)$ 

Say Trout weight is normally distributed with mean 2kg, std dev. .3 kg.

What is the prob a random trout weighs 2.5 kg or less?

"mass"

X = weight in kg $\sim N\left(\underset{\mu}{2}, \underset{\sigma^2}{(.3)^2}\right)$



$P(X \le 2.5)$ = normalcdf(~~-10000~~, 2.5, 2, .3)

$\approx 95\%$

Quantiles / Percentile

What is the 90th percentile of trout?

ie what weight are 90% of trout less than?

what x is going to satisfy $P(X \le x) = .9$

Find z for $P(Z \le z) = .9$

"

$z_{.9}$ ie $z_\alpha \ni P(Z \le z_\alpha) = \alpha$

From Table $z_{.9} = 1.28$ ie $P(Z \le 1.28) = .9$

$z = \frac{x - \mu}{\sigma}$ so $x = \mu + \sigma \cdot z$

$x_{(.9)} = 2 + (.3)(1.28)$

$= 2.384$

invNorm(.9)

invNorm(α) ⇐ std Normal

invNorm(α, μ, σ) ⇐ $X \sim N(\mu, \sigma^2)$

use $\Phi(z) = P(Z \leq z)$

z_α = the z satisfying $P(Z < z_\alpha) = \alpha$