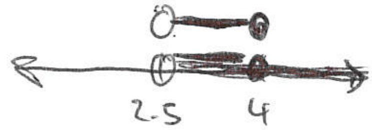


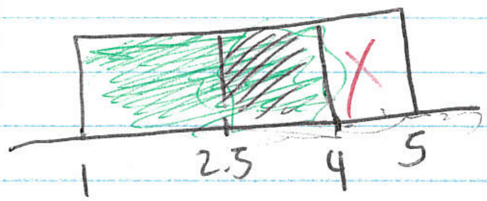
$X \sim Unif_c(1,5)$ $\qquad$ $X > 2.5$ and $X \leq 4$

$$P(X > 2.5 \mid X \leq 4) = \frac{P(2.5 < X \leq 4)}{P(X \leq 4)} = \frac{\frac{1.5}{4}}{\frac{3}{4}}$$

$\uparrow$

$P(\cancel{X}\ 1 \leq X \leq 4$

$$= \frac{1.5}{3} = \frac{1}{2}$$



$Y \sim Unif_c(1,4)$

$P(Y > 2.5) = \frac{1}{2}$

$P(X > 3.5 \mid X \leq 4)$

$$\frac{P(3.5 < X < 4)}{P(X < 4)} = \frac{.5/4}{3/4}$$

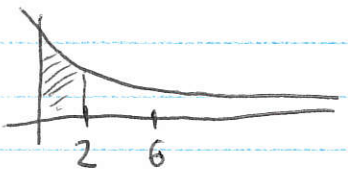
$$= \frac{.5}{3} = \frac{1}{6}$$

Say waiting time at a DMV line follows an exponential distribution with mean waiting time of 6 minutes.

What is the probability that it takes fewer than 2 minutes to help a customer?

$X \sim exp(6)$

$$f(x) = \begin{cases} \frac{1}{6} e^{-\frac{x}{6}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$



$$\int_0^2 \frac{1}{6} e^{-x/6}\, dx = \left[-e^{-x/6}\right]_0^2 = -e^{-\frac{1}{3}} + 1$$

Let $u = \frac{x}{6}$ $du = \frac{dx}{6}$

$$\int_0^2 \frac{1}{6} e^{-x/6} dx = \int_0^{1/3} e^{-u}\, du = -e^{-u}\Big|_0^{1/3}$$

$$= -e^{-\frac{1}{3}} + 1 = .2835$$

Gamma (α, β)

The amount of time it takes for 5 customers follows a Gamma distribution with $\alpha = 5$, and $\beta = 6$

~~wh~~ 5 people are in front of you,
What is the probability you'll be at the counter in less than 30 min?

$$X \sim \text{Gamma}(\overset{\alpha}{5}, \overset{\beta}{6})$$

$$P(X < 30) = \int_0^{30} \frac{1}{6^5 \Gamma(5)} x^4 e^{-x/6} dx$$

$$u = \frac{x}{6} \quad du = \frac{dx}{6}$$

$$= \int_0^{5} \frac{1}{\Gamma(5)} u^4 e^{-u} du$$

$$F(\underset{x}{5}; \underset{\alpha}{5}) = .5595$$

$$1 - \text{poissoncdf}(\underset{\substack{\uparrow \\ \text{"}x\text{"}}}{5}, \underset{\substack{\uparrow \\ \alpha-1}}{4})$$

If X_1, X_2 independent,

$$X_i \sim \text{Gamma}(\alpha_i, \beta)$$

$$X_1 + X_2 \sim \text{Gamma}(\alpha_1 + \alpha_2, \beta)$$

---

say $X_1, \dots, X_n$ indep $\exp(\beta) \equiv \text{Gamma}(1, \beta)$

$$\sum_{i=1}^{n} X_i \sim \text{Gamma}(n, \beta)$$

If X_1, X_2 are Normal, independent

$$X_i \sim N(\mu_i, \sigma_i^2)$$

$$X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$$

$\Rightarrow$ $X_1, \dots, X_n$ indep Normal $X_i \sim N(\mu_i, \sigma_i^2)$

$$\sum X_i \sim N(\sum \mu_i, \sum \sigma_i^2)$$

Ch 8 Sampling Distributions

Sampling from population

Population $\longleftrightarrow$ Probability Distribution.

sample size n - each observation is independent
$x_1, \dots, x_n$ (ie, random) and all from same population,

we say sample is iid

independent and indentically distributed.

A statistic is any function of random sample data.

· Sample mean $\bar{X} = \dfrac{X_1 + \cdots + X_n}{n}$

sample median $\tilde{X} = \begin{cases} X_{(n+1/2)} & n \text{ odd} \\ \dfrac{X_{(\frac{n}{2})} + X_{(\frac{n}{2}+1)}}{2} & n \text{ even} \end{cases}$

sample mode value most common

Take $X_1, \dots, X_n$ iid ~~with~~ distribution with mean μ variance σ^2 eg $E(X_i) = \mu$

$$E(\bar{X}) = E\left(\frac{1}{n}(X_1 + \cdots + X_n)\right)$$
$$= \frac{1}{n} E(X_1 + \cdots + X_n) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}\sum_{i=1}^{n}\mu$$
$$= \frac{1}{\cancel{n}}\cancel{n}\mu = \mu$$

- $E(\bar{X}) = \mu$ regardless of type of distribution

- $Var(\bar{X}) = Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n^2} Var\left(\sum_{i=1}^{n} X_i\right)$ X_i's indep

$$= \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i)$$

$$= \frac{1}{n^2}\sum_{i=1}^{n} \sigma^2 = \frac{1}{n^2} n \cdot \sigma^2$$

$$= \frac{\sigma^2}{n}$$