Key

Quiz 10 STAT 381, APPLIED STATISTICAL METHODS I, SPRING 2015

NAME:

Consumers have recently been complaining that the bulbs sold by Acme Electric are unusually dim. A random sampling of 25 "60 watt" bulbs were tested, and the average wattage was actually 58.2 watts with a standard deviation of 1.8. We wish to determine if the consumers are correct.

a) (2 points) State the null and alternative hypotheses.

$H_0: \mu = 60$

$H_1: \mu < 60$

b) (2 points) Assuming the population is normal, what is the value of the test statistic, and what is its distribution (include parameter value(s))?

$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{58.2 - 60}{1.8/\sqrt{25}} = -5$ t-distribution with 24 d.f.

c) (2 points) What rejection region would you use at the significance level of $\alpha = .05$?

$t < -1.711$

d) (2 points) What is the P-value of the test statistic?

$P(T < -5) = .0000207$

From table we can say it is $< .001$

e) (2 points) What is your conclusion using $\alpha = .05$?

There is strong evidence to reject H_0. The bulbs are dim

Bonus (6 points) If in fact the true $\mu = 57$ watts, what is the power of this test (using $\alpha = .05$)?

Hint: Convert the critical value of T to a critical value of $\bar{X}$, and find the probability that $\bar{X}$ falls into the rejection region when $\mu = 57$.

$t < -1.711 \Rightarrow \frac{\bar{x} - 60}{1.8/5} < -1.711 \Rightarrow \bar{x} < 59.38404$

if $\mu = 57$, ~~$P(\bar{X} < 57) = .5$~~

$P(\bar{X} < 59.38404) = P\left(\frac{\bar{x} - 57}{1.8/5} < \frac{59.38404 - 57}{1.8/5}\right)$

$= P(T < 6.6223) = .99999$!!!