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Pairing Up

Shalini is a TA for Integral Calculus. She runs discussion sections in which material from lecture is reviewed and practiced. In a typical session, she answers questions for about 10 minutes, then proposes several exercises to be worked by the students during the period. Shalini circulates and helps where needed, encouraging group discussion. She writes on the board only if there is widespread confusion.

Lately, Shalini has felt that the students are relying too heavily on her. She decides to have them work in pairs and explain things to each other, in the hope that it will make them think more for themselves.

Shalini: Today, I want everyone to work with a partner. If you don't know anyone, I'll match you up. You should work on the problem that I'm going to put on the board, and it's really important that both you and your partner always know what's going on. Only ask me questions as a last resort. Questions?

Maria: Do we have to hand it in?

Shalini: No, this is just like any other day, except that you must work together.

She writes the following on the board:

A cone of height 8 ft and top radius 3 ft is filled with liquid to a depth of 6 ft. Assuming the liquid weighs 10 lb/ft³, how much work will it take to remove all the liquid out of the top?

Shalini: OK, pair up.

She wanders around for a bit, giving students a chance to settle. Noticing that Evan, a sullen but high achieving student, is about to work alone, Shalini taps on his book and prods, "C'mon, find a partner. Chris, why don't you come work with Evan." Chris is also bright, and more laid back than Evan. Shalini thinks they might complement each other's strengths, and perhaps Evan will recognize the benefits of collaboration.

Shalini is pleased to see that Cassandra and Rae have paired up. Both are quiet, and struggle with the material. Rae often works with her friend Jamie, who is absent today. Jamie has a solid background, and Rae always defers to her, mostly copying Jamie's solutions.

Shalini: How are we doing here?

Shalini: Why don't you just start by drawing a picture of the problem? See how much you can label on it. I'll come back in a little while.

Shalini sees Chris and Evan working separately. She goes over, hoping some cheery encouragement will make them communicate.

Shalini: So, where are you two at? Chris, do you agree with what Evan has done so far?

Chris: (guiltily) Uh, we're just working it out first for ourselves, and then we'll compare answers.

Shalini: OK. But today I'd really like everyone to work together. So, Evan, why don't you try and explain your approach to Chris?

Evan: I don't have an approach.

Shalini: That's fine. I just meant—what do you have so far? It looks like a good start. (She sees that he has the basic setup correct.)

Evan: I don't have anything. How do I do this problem? If you would tell me, then I could get it.

Shalini: I think we should start with what you've got. Explain to me what these variables mean? (She points to his equation.)

Evan: Don't you know?

Shalini: I'd like you to tell me.

Evan: You know, I could be this confused at home. I don't need to come here for this.

Chris is frowning at a neat diagram of the problem, but looks up in surprise upon hearing this.

Chris: Gee, I thought you knew how to do all the problems.

Evan: I can do the problems, I just don't understand what they mean.

Chris: Hmm, I have the opposite problem. Let me look at your figure ... your equation's different from mine ... oh, I see, you're measuring from the top, not the bottom.

Evan: Yeah, I see ... it would make the equation easier if I did it the other way around.

Shalini, relieved at not having to respond to Evan's rudeness, leaves them at it and goes across the room to help Trina and Lucas, who have been waving frantically at her.

Trina: (cheerfully) Is this right?

Shalini: Why don't you tell me about it?

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Lucas: We split up the cone into pieces, and then integrated.

Shalini: Walk me through it.

Lucas: We made it into circles, and used $\text{Work} = (\text{force})(\text{distance})$. Since the force is 10, and the distance is x , we got

$$\int_0^6 10x \, dx = 180.$$

Shalini: Trina, do you think it's right?

Trina: It sounds good to me.

Shalini is flustered—she doesn't want to outright say it is wrong, as that will defeat the collaborative approach. But she doesn't want to be the TA whose students have a book full of wrong answers, either.

Shalini: So, you said that you sliced the cone up?

Lucas: Yes.

Shalini: Could you tell me how you used the slices to come up with the integral?

Both students seem confused, and laugh. Trina starts to flip in her textbook.

Shalini: I want you to draw me a slice, and figure out the work to move just that piece. Then see how you can use it in your integral.

Trina: So we're wrong?

Shalini: I didn't say that. I just want us to see where the integral comes from. That way we can tell if it's right.

Shalini leaves Trina and Lucas unsure of what they are looking for, visits some other pairs, and then heads back, as promised, to Cassandra and Rae.

Shalini: How's it going?

Cassandra: We drew a picture. (She points to her page.)

Shalini: OK, what's next.

Rae: Divide it up?

Shalini: Why might we do that?

Rae: Because that's how you always do these problems.

Rae draws some horizontal lines across the cone.

Shalini: Let's draw a typical slice over to the side. (She waits, and when neither student moves, draws one herself.)

Shalini: What is the radius of this slice?

Rae: 3 ft?

Shalini: If I chose a different slice, what would its radius be?

Cassandra's face starts to light up.

Cassandra: It would be different! The radius has to do with where the slice is. It has to do with x ! Rae, do you get it?

Suddenly, Shalini sees that people are packing up, and realizes that class is over. She makes a quick announcement, encouraging people to complete the problem, and bring questions next time. She worries that Rae and Cassandra have just begun to understand, but are nowhere near to being able to complete this (or any other) problem on their own. She makes a mental note to talk to Trina and Lucas next time, hoping that they will have discovered their own errors. Shalini cannot decide if she has fulfilled her job as TA today—the only students that she knows will solve the problem are Evan and Chris, and she is not convinced that she has played any part in their success. She hopes that in some capacity she has motivated her students to think independently, but wonders if they will benefit from this skill during the course. The students will need to study this material for the next examination . . . perhaps she should just spend the time showing them how to do the problems.