CAREER: Realizing derived equivalences

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1 Education Proposal

At Purdue, my teaching has centered on linear algebra courses for engineering and science undergraduates. Linear algebra naturally requires the student to reason at a level deeper than just following an algorithm for producing numerical answers. For the students who find these challenges frustrating or difficult, I provide encouragement and my confidence that time and effort often lead to understanding. In turn, my effort is rewarded whenever I witness a student's renewed engagement in learning or spark of excitement at mastering new material. For this work, I was chosen by the administration as the 2001 School of Science Outstanding Assistant Professor for contributions to undergraduate teaching.

With an NSF Career award, I plan to expand my involvement with course development, undergraduate and graduate research, and mentoring women.

1.1 Undergraduate research and recruitment

I have been invited to participate in the University of Chicago REU program in the summer of 2002. This is a large program (35 students in 2001) with many students who will go on to graduate school. I will teach for two weeks (12 hours) on a specific topic in algebra or algebraic topology. The emphasis will be on introducing problems for the students to try which are either open or new in some way, such as calculational examples or experiments.

The Purdue REU program is currently smaller and based on individual research projects. I hope to import some of the interactive aspects of the UC program into the Purdue REU program. In the following summers I will supervise research projects on knot theory, computer programs for computing free resolutions which appear in my research, or other algebraic or topological problems based on the interests and experience of the students who apply to the program.

I have given a talk on "Knot theory and DNA" based primarily on Sumners work [Sum95] to several audiences. Although this talk is very accessible, it imparts the true flavor of algebraic topology by explaining the use of polynomials to differentiate knots. I will continue to give this talk to undergraduates during my visits to other colleges and universities as part of Purdue's VIGRE program to recruit graduate students in mathematics. I believe this is a good recruitment tool because it covers different material than the usual curriculum and I use actual hands-on models of knots.

1.2 Graduate teaching and research

I actively mentored several University of Chicago graduate students, including Michael Mandell, Maria Basterra, Dan Isaksen and Laura Scull. I was on the thesis committee for Maria

Basterra and for Purdue University topology graduate student Michael Jackson. I continue to visit the University of Chicago regularly and often interact with current students including Halvard Fausk, Ben Blander and Rochelle Pereira. I have also been mentoring and working with Dan Dugger, a post-doc at Purdue. To aid the recruitment of students into algebraic topology at Purdue, I plan to give several introductory seminars in the graduate student seminar and the regular topology seminar, and to teach some of the courses in the graduate topology sequence in the next few years.

With L. Avramov, I plan to organize a workshop or summer school for graduate students and other young researchers on derived categories and related topics of common interest in the fields of Algebraic Topology, Representation Theory, Group Cohomology, Commutative Algebra and Algebraic Geometry. Specific topics will be selected from: the Alperin and Broué conjectures [Alp87, Bro90], the axiomatic treatment of stable homotopy theories and Bousfield localization from [HPS97], homotopical algebra [Qui67], differential graded homological algebra [AFH] and motivic homotopy theory [Voe98], with the goal of increasing research interactions and applications in these fields. To take advantage of the expertise located at Purdue in these areas, we plan to hold this workshop at Purdue in the summer of 2004. Students and young researchers from foreign countries or who are members of underrepresented minority groups will be especially encouraged to attend this workshop. Additional funds will be sought to support their attendance. My next step will be to recruit speakers and narrow the possible topics related to derived categories. I have received an enthusiastic preliminary response from D. Benson. Pending funding, this workshop would take place over two weeks with each week devoted to two lecture series and a four day conference would be held at the end of these two weeks. My related experiences include helping mentor students at the month-long IMA Summer Program in Algebraic Topology at the University of Chicago in 1997 and co-organizing the two day Midwest Topology Seminar at Purdue for the fall of 2001.

1.3 Undergraduate course development

Next year I plan to teach a new course "Theory of equations." The development of this course is joint work with J. McClure, our department's current undergraduate chairman, and G. Harel, a professor of mathematics education at the University of California, San Diego. The goals of this course are to help our math education students master skills in algebra, develop confidence for teaching algebra, and provide motivation for the material contained in an abstract algebra course. We expect these prospective teachers to find this study of polynomials both more accessible and more relevant than the current abstract algebra courses. Based on McClure's and others experience using original texts [McC00, LaPe94, LaPe98], we plan to use Euler's "Elements of Algebra" [Eul84] as a main text to spark interest and to emphasize that this course is not just revisiting high school algebra. We will also use challenging problems from [Chr69], [Gel93] and [PS96] to emphasize conceptual thinking rather than symbol manipulation. The current plan is to first offer this course during a month-long intensive program in May as enrichment to our regular academic year curriculum. The department offered one course during this period for the first time this year and plans to continue to develop courses for this program. We then plan to continue to develop this course for future years.

Another continuing project, for a linear algebra course for engineers which I have taught several times, is to collect successful interactive demonstrations, homework exercises and group projects which use the Matlab computational package. This collection could then be used in the 6-10 sections per semester, many of which are led by young teachers who have not taught linear algebra before.

1.4 Mentoring women

As one of the relatively few women who have made it (part-way) through the pipeline, I plan to continue to support and encourage women's advancement in mathematics. As part of the Women in Science Program (WISP) at Purdue I will attend monthly meetings of the Graduate Mentoring Program. I have also volunteered to be a Faculty Fellow in the WISP Residential Mentoring Program for a dorm floor of undergraduate women in science. Since this program began in 1997, these women have been retained in the School of Science at a much higher rate than a matched control group. If appropriate, I would also use part of this grant to support the WISP Travel Grants for Professional Meetings for graduate and upper-level undergraduate students to cover the cost of travel to present papers or posters.

I will also rejuvenate a mentoring program within the department of mathematics. I will co-organize three or four teas each year for the female graduate students and faculty. Occasionally, these will be scheduled to coincide with a visit from a woman mathematician (e.g., a speaker in one of our research seminars) who would be invited to the tea to give a short talk on her experiences and her research. Occasionally, I will also invite the female undergraduate math majors to join us. If there is interest, I would also encourage and support the students in organizing other activities such as the "Expanding your Horizons" workshops on math and science for girls in Indiana or visits from women mathematicians outside academia. With these programs, I will use other successful programs as guides such as the Carnegie Mellon University Women in Computer Science program.

I am currently a mentor in the Association of Women in Mathematics Mentor Network. As a post-doc at the University of Chicago, I helped organize teas for the female graduate students three times per year, brown-bag lunch discussions and an annual spring meeting of the female undergraduate and graduate students. As a graduate student at MIT, I organized weekly brown-bag lunches and helped initiate a regular mathematical seminar given by women mathematicians. As an undergraduate, I co-organized the Harvard/Radcliffe Women in Science group which had regular study breaks and dinner meetings with women faculty.

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