

Math320: Final Exam
Spring 2008

Problem 1. (30 pts) Diagonalize the following matrix. (Find X and D such that $A = XDX^{-1}$.)

$$A = \begin{pmatrix} +1 & -1 & 2 \\ -3 & -1 & 6 \\ -1 & -1 & 4 \end{pmatrix}$$

Problem 2. (30 pts) Find all values of c for which A is diagonalizable.

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & c & 0 \\ 5 & 1 & 1 \end{pmatrix}$$

Problem 3. (35 pts) Let A be a 2×2 matrix. Suppose that $1, -1$ are the eigenvalues of A and $u_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $u_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ are the corresponding eigenvectors of A .

- a) Find $\det A$, $\det A^6$.
- b) Find B such that $B^3 = A$.
- c) Compute e^A .

Problem 4. (40 pts) Let

$$S = \text{Span} \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} \right\} \text{ and } b = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}.$$

- a) Find a basis of S . What is the dimension of S ?
- b) Use the Gram-Schmidt process to find an orthonormal basis of S .
- c) Find $\text{Rep}_G b$, where G is the orthonormal basis from part b).

Problem 5. (30 pts) Let $V = \text{Span}\{\cos x, \sin x\}$. S is a subspace of the space of differentiable functions C^∞ . Let D be the differentiation operator: $D(f) = f'$.

- a) Show that $D: V \rightarrow V$ is a linear map.
- b) If $E = \{\cos x, \sin x\}$ is the fixed basis of V , find $\text{Rep}_{E,E} D$.

Problem 6. (35 pts) Let $A = \begin{pmatrix} 3 & 1 & 2 \\ 3 & 5 & 7 \\ 1 & -1 & 1 \\ 6 & 7 & 8 \end{pmatrix}$ and $REF(A) = \begin{pmatrix} 3 & 1 & 2 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. Suppose

that $L: \mathbb{R}^3 \rightarrow \mathbb{R}^4$ is a linear map given by $L(x) = Ax$.

- a) Find $\ker(L)$.
- b) Find $\text{Range}(L)$.
- c) Find $\text{rank}(L)$ and $\text{nullity}(L)$.
- d) Is L an isomorphism? (Explain.)