Sample Exam 2

Problem 1. Let $\mathbf{u_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\mathbf{u_2} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\mathbf{v_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\mathbf{v_2} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

a) Find the transition matrix S corresponding to the change of basis from $\{\mathbf{u_1}, \mathbf{u_2}\}$ to $\{\mathbf{v_1}, \mathbf{v_2}\}$.

b) If $L : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear operator such that $L(\mathbf{v_1}) = \mathbf{v_1} - 2\mathbf{v_2}$ and $L(\mathbf{v_2}) = -\mathbf{v_1} + 2\mathbf{v_2}$, find the matrix representation of L with respect to the basis $\{\mathbf{v_1}, \mathbf{v_2}\}$.

c) Find the kernel and image of L.

c) Find the matrix representation of L with respect to the basis $\{\mathbf{u_1}, \mathbf{u_2}\}$.

Problem 2. Let $A = \begin{pmatrix} -2 & 0 & 0 \\ -5 & 3 & 0 \\ -5 & 2 & 1 \end{pmatrix}$.

a) Find $det A^5$.

b) Find A^{-1}.

c) If the map $L : \mathbb{R}^3 \to \mathbb{R}^3$ is given by $L(x) = Ax$ for any $x \in \mathbb{R}^3$, show that L is a linear transformation.

d) Is L one-to-one? Is L onto? Is L an isomorphism?