Math 320 Midterm Exam February 20, 2009

Write your NAME on your exam booklet.

Write the solutions to the **five** problems in the exam booklet and clearly indicate which problem you are solving. You are expected to abide by the University's rules concerning academic honesty.

1. Show that the set $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ spans $\mathbb{R}^3$.

2. No justification is necessary for any part of this problem.

(a) Find a basis for the row space of $A = \begin{pmatrix} 1 & 0 & -3 & 0 & 5 \\ 0 & 1 & 4 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{pmatrix}$.

(b) Find the nullspace of A. (Recall, the nullspace of A is the solution set of the homogeneous system of equations with coefficient matrix A.)

(c) Find a basis and the dimension of the nullspace of A.

3. (a) Find a basis for the subspace of 2 by 2 matrices given by $W = \{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ such that $a + c = 0\}$.

(b) Prove that this is a basis for W.

4. Answer each of the following TRUE or FALSE. If TRUE, explain why; if FALSE give a counter example.

(a) If $\{v, w, x\}$ is a linearly dependent set, then $\{v, w\}$ is also linearly dependent.

(b) If $\{v, w, x\}$ spans V, then $\{v, w, x, y\}$ also spans V.

(c) A linear system of three equations and five unknowns always has a solution.

5. (a) Is $W = \{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + a \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$ such that $a, b \in \mathbb{R}\}$ a vector space?

(b) If yes, find a basis. If no, show it is not a vector space.