

Solutions for Quiz on 1/16

1. Use Gauss (or Gauss-Jordan) method to find the solutions to the following system of equations:

$$x + 2y = 8$$

$$3x + 4y = 14$$

$$-x - y = -3$$

Using matrix notation : $\left(\begin{array}{cc|c} 1 & 2 & 8 \\ 3 & 4 & 14 \\ -1 & -1 & -3 \end{array} \right)$

$-3(\rho_1 + \rho_2)$ and $\rho_1 + \rho_3$ gives: $\left(\begin{array}{ccc} 1 & 2 & 8 \\ 0 & -2 & -10 \\ 0 & 1 & 5 \end{array} \right)$

Swap row 2 and row 3: $\left(\begin{array}{ccc} 1 & 2 & 8 \\ 0 & 1 & 5 \\ 0 & -2 & -10 \end{array} \right)$

$2\rho_2 + \rho_3$ gives: $\left(\begin{array}{ccc} 1 & 2 & 8 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{array} \right)$

Using Gauss' method, in equations we then have:

$$x + 2y = 8$$

$$0 + y = 5$$

$$0 = 0$$

Solving gives $y = 5$ and $x + 10 = 8$ so $x = -2$.

Using Gauss-Jordan method instead, continuing with the matrices $-2\rho_2 + \rho_1$ gives: $\left(\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{array} \right)$

So $x = -2$ and $y = 5$.

In vector notation we have: $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$

Note that $-2 + 2(5) = 8$

$$3(-2) + 4(5) = 14$$

$$-(-2) - (5) = -3$$

2. The second question asked for two systems of equations each with four variables and three equation: one in echelon form and one not in echelon form.

There are many correct answers to this, of course. I'm recycling from above:

a) In echelon form:

$$x + 2y + z + w = 8$$

$$4y = 14$$

$$z = -3$$

b) Not in echelon form:

$$x + 2y + z + w = 8$$

$$3x + 4y = 14$$

$$-x - y = -3$$