Solutions for Quiz on 2/6

1. Define span $S = [S]$ the span of the set $\{\vec{s_1}, ..., \vec{s_n}\}$.

The span of S is the set of all linear combinations of the elements of S over the field of scalars $\mathbb{F}$. That is, $[S] = \{a_1\vec{s_1} + \cdots + a_n\vec{s_n} \mid a_i \in \mathbb{F}\}$.

2. Is $\{1 + x + 2x^2, 2 + 3x + 5x^2, 3 + 4x + 7x^2, 4 + 5x + 9x^2\}$ linearly independent in P_2?

Answer: **No.**

P_2, the vector space of quadratic polynomials in x over the reals, has previously been shown to be three-dimensional. The given set has four polynomials. Since $4 > 3$, the set cannot possibly be linearly independent. In fact, the third polynomial is the sum of the first two.

A blind approach to the problem could use matrix reduction. Naturally associate each polynomial to a row vector in $\mathbb{R}^3$ and concatenate them into a matrix:

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \\ 3 & 4 & 7 \\ 4 & 5 & 9 \end{pmatrix}$$

Since this matrix has more rows than columns, any echelon form of the matrix will have a zero row. In fact, the RREF is:

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

This matrix does have zero rows, as expected. Therefore the rows of the original matrix were not linearly independent. (The same method can be used with column vectors instead, yielding the same result.)