Math 547 **Final** **Due December 7, 2007**

1. Let T be the standard torus. Find a CW complex X such that $H_n(X) \cong H_n(T)$ for all n, but T and X are not homotopy equivalent. (20pts)

2. Let X be the space obtained by gluing two 3-spheres along their equators. Using any method, compute the homology of X. (20pts)

3. Recall that the 3-sphere S^3 is homeomorphic to the group $SU(2)$. Suppose I is a discrete subgroup of $SU(2)$; it acts on $SU(2)$ by left multiplication (and therefore it acts on S^3.) Let $P = S^3/I$ be the orbit space of this action. (40pts)

(a) Give a short argument that the natural map $S^3 \to P$ is a covering map. (You may use all results about covering spaces we proved.)

(b) Identify the fundamental group of P.

(c) Assume I is a non-abelian simple group. Compute $H_1(P)$.

(d) Show that P is a smooth manifold.