Math 547 **Homework 7** **Due November 26, 2007**

1. Let X and Y be two surfaces (for this problem, a surface is a compact connected topological manifold of dimension 2). The connected sum $X\#Y$ of X and Y is the new surface obtained by cutting out an open disc from both X and Y and then gluing X and Y along the boundary circles of those discs. (20pts)

a) Let X be any surface. Show that $H_*(X\#S^2) \cong H_*(X)$.

b) Let X_n be the connected sum of n tori. Using the Mayer-Vietoris sequence, compute the homology $H_*(X_n)$.

c) Compute the homology $H_*(X_n\#\mathbb{R}\mathbf{P}^2)$.

2. Describe $\mathbb{R}\mathbf{P}^n$ as a CW-complex with one cell in each dimension $0 \leq n$. Compute $H_*(\mathbb{R}\mathbf{P}^n)$. Let $\mathbb{R}\mathbf{P}^\infty$ be the union of the spaces $\mathbb{R}\mathbf{P}^n$ (a CW-complex with one cell in each dimension $n \geq 0$.) Calculate $H_*(\mathbb{R}\mathbf{P}^\infty)$. (20pts)

3. Let X be a connected CW-complex with two 0-cells, one 1-cell, one 3-cell and two 8-cells. Compute the homology $H_*(X)$. (10pts)

4. Use the Mayer-Vietoris sequence to compute the homology of the space obtained by connecting the North and South poles of a 2-sphere by an interval. (10pts)

5. Let L be a compact connected topological manifold of dimension one. Prove that L is homeomorphic to S^1. (10pts)