1. MATH 494 PAPER TOPICS

Topic 1.1. The Mordell-Weil Theorem. This theorem says that the group of rational points on an elliptic curve is a finitely generated abelian group. Chapter 3 of the book Silverman and Tate, Rational Points on Elliptic Curves, Springer, 1992 is a good place to learn about this theorem.

Topic 1.2. The Nagell-Lutz Theorem. This theorem describes the points of finite order in the group of $\mathbb{Q}$-rational points on an elliptic curve. Chapter 2 of the book Silverman and Tate, Rational Points on Elliptic Curves, Springer, 1992 is a good place to learn about this theorem.

Topic 1.3. Fermat's descent argument and showing that certain Diophantine equations have no solutions. Chapter XIII of Hardy and Wright, An introduction to the theory of numbers, Oxford Science Publications, 1996 is a good place to start.

Topic 1.4. Poncelet's Theorem. This theorem studies polygons that are inscribed and circumscribed in a pair of conics. You can learn more about it in the paper of Griffiths and Harris, A Poncelet's Theorem in space, Comment. Math. Helvetici 52, 145-160 (1977)

Topic 1.5. The Alexander-Hirschowitz Theorem for points in the plane. This theorem answers the question: When do general double points impose independent conditions on curves of degree d? A good place to start is the paper by Brambilla and Ottaviani "On the Alexander-Hirschowitz Theorem".

Topic 1.6. A Littlewood-Richardson rule for the Grassmannian. Littlewood -Richardson rules are rules for multiplying Schubert cycles in the Grassmannian. A good place to start is Fulton's book on Young Tableaux.

Topic 1.7. Tsen's Theorem. This theorem says that if you have a polynomial of degree $d < n$ in n variables with coefficients in $\mathbb{C}[t]$, then it has a solution over $\mathbb{C}[t]$. A good place to start is Shafarevich's Basic Algebraic Geometry 1, Chapter I.6.

Topic 1.8. The Cayley-Bacharach Theorem. In its simplest form, this theorem says that any cubic containing 8 of the points of intersection of two cubics contains the ninth. A good place to start is the paper by D. Eisenbud, M. Green and J. Harris on CayleyBacharach theorems and conjectures in the Bulletin of the AMS, 1996

Topic 1.9. Plane curve singularities and torus knots. If one takes the intersection of a curve with a small sphere around a unibranch singularity, one gets an iterated torus knot. To learn more about this subject see Brieskorn and Knorer's Plane Algebraic Curves, Birkhauser, 1981.

Topic 1.10. Resolving plane curve singularities using Puiseux series. Plane curve singularities can be resolved using the technique of Puiseux series (which are power series that have some fractional expoents). To learn more about this subject see Brieskorn and Knorer's Plane Algebraic Curves, Birkhauser, 1981.

Topic 1.11. Elliptic functions. These are meromorphic functions on complex one dimensional tori. They are very closely related to elliptic curves. You can learn more about them in Ahlfors, Complex Analysis, Chapter 7.

Topic 1.12. The geometry of cubic surfaces. Cubic surfaces have 27 lines that intersect in a beautiful pattern. Igor Dolgachev's lecture notes on Classical Geometry have a large amount of material on cubic surfaces.