1. MATH 494: Homework 3

This problem set is due Wednesday November 17. You may work on the problem set in groups; however, the final write-up must be yours and reflect your own understanding.

Problem 1.1. Show that the wedge

$$v = -e_1 \wedge e_2 + e_1 \wedge e_3 + e_1 \wedge e_4 + e_2 \wedge e_3 + 2e_2 \wedge e_4 - e_3 \wedge e_4$$

is decomposable. Find two vectors w_1, w_2 such that $v = w_1 \wedge w_2$.

Problem 1.2. Find all the Plücker relations for $G(2,5)$. Determine whether the wedge

$$e_1 \wedge e_2 + e_3 \wedge e_4 + e_1 \wedge e_3 + e_2 \wedge e_4 + 2e_1 \wedge e_5 + e_3 \wedge e_4 + e_1 \wedge e_4 + 2e_3 \wedge e_5$$

is completely decomposable.

Problem 1.3. List all the Schubert classes in $G(2,5)$. Describe the set of lines in $\mathbb{P}^4$ that they parameterize. Calculate the intersection table for the cohomology of $G(2,5)$.

Problem 1.4. Show that a general hypersurface of degree $d > 2n-3$ in $\mathbb{P}^n$ does not contain any lines.

Problem 1.5. Show that a quadric hypersurface in $\mathbb{P}^4$ of corank 1 (i.e., a quadric projectively equivalent to $x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0$) has a 1-dimensional family of planes. Show that a smooth quadric hypersurface in $\mathbb{P}^4$ does not contain any planes. Show that even though the incidence variety

$$I = \{(P,Q) \mid P \in \mathbb{G}(2,4), Q \text{ a quadric hypersurface}, P \subset Q\} \subset \mathbb{G}(2,4) \times \mathbb{P}^{14}$$

has dimension 14, the second projection to $\mathbb{P}^{14}$ is not surjective.

Problem 1.6. Show that the locus of hypersurfaces of degree d in $\mathbb{P}^n$ that are singular is an irreducible hypersurface in $\mathbb{P}^{\binom{n}{d}-1}$. This hypersurface is known as the discriminant hypersurface. (Extra credit: Show that the degree of the discriminant hypersurface is $(n+1)(d-1)^n$.)