Math 121 – Quiz 3 Solution

1. Construct a rational function $R(x)$ that has the following properties:
   - $x = 1$, $x = 2$, and $x = 3$ are vertical asymptotes
   - $y = 0$ is a horizontal asymptote
   - the x-intercept is at $x = -1$
2. Solve the inequality $\dfrac{x-4}{2x+4} \geq 1$.

Solution:

1. Since $x = 1$, $x = 2$, and $x = 3$ are vertical asymptotes, we know that:

$$R(x) = \frac{p(x)}{(x-1)(x-2)(x-3)}$$

   Since $y = 0$ is a horizontal asymptote, the degree of $p(x)$ is less than 3. Also, since the x-intercept is at $x = 0$ we know that $p(-1) = 0$. Therefore, we can say that $p(x) = x + 1$. So, the function $R(x)$ is:

$$R(x) = \frac{x+1}{(x-1)(x-2)(x-3)}$$

2. Rewriting the inequality, we have:

$$\begin{aligned} \frac{x-4}{2x+4} &\geq 1 \\ \frac{x-4}{2x+4} - 1 &\geq 0 \\ \frac{x-4-(2x+4)}{2x+4} &\geq 0 \\ \frac{-x-8}{2(x+2)} &\geq 0 \end{aligned}$$

   Using the fact that the zeros of the numerator and denominator of $f(x) = \dfrac{-x-8}{2(x+2)}$ are $x = -8, -2$, we set up the following table:

Interval	$(-\infty, -8)$	$(-8, -2)$	$(-2, \infty)$
Number Chosen	-9	-3	0
Value of f	$f(-9) = -\frac{1}{14}$	$f(-3) = \frac{5}{2}$	$f(0) = -2$
Location of graph	below x-axis	above x-axis	below x-axis

   Since $f(x) \geq 0$, the solution is $-8 \leq x < -2$.