Math 121 – Quiz 5 Solution

Given

$$\cos\theta = \frac{2}{3}, \quad 0 < \theta < \frac{\pi}{2} \qquad \text{and} \qquad \sin\beta = \frac{3}{5}, \quad \frac{\pi}{2} < \beta < \pi$$

compute the following:

(a) $\cos 2\theta$ (b) $\cos(\theta - \beta)$ (c) $\sin(\theta + \beta)$

Solution:

1. $\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{2}{3}\right)^2 - 1 = -\frac{1}{9}$
2. $\cos(\theta - \beta) = \cos\theta\cos\beta + \sin\theta\sin\beta = \left(\frac{2}{3}\right)\left(-\frac{4}{5}\right) + \left(\frac{\sqrt{5}}{3}\right)\left(\frac{3}{5}\right) = \frac{3\sqrt{5} - 8}{15}$
3. $\sin(\theta + \beta) = \sin\theta\cos\beta + \cos\theta\sin\beta = \left(\frac{\sqrt{5}}{3}\right)\left(-\frac{4}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{3}{5}\right) = \frac{6 - 4\sqrt{5}}{15}$