Math 121 – Section 4.2 Solutions

11. The domain of $R(x) = \dfrac{4x}{x-3}$ is all real numbers x except $x = 3$.

15. The domain of $F(x) = \dfrac{3x(x-1)}{2x^2-5x-3}$ is all real numbers x except $x = -\dfrac{1}{2},\ 3$.

18. The domain of $F(x) = \dfrac{x}{x^4-1}$ is all real numbers x except $x = \pm 1$.

24. Using the given graph of $y = f(x)$, we have:
    (a) the domain of $f(x)$ is all real numbers x except $x = -1$; the range of $f(x)$ is $y > 0$
    (b) the y-intercept is $y = 2$; there are no x-intercepts
    (c) $y = 0$ is the horizontal asymptote
    (d) $x = -1$ is the vertical asymptote
    (e) there is no oblique asymptote

27. Using the given graph of $y = f(x)$, we have:
    (a) the domain of $f(x)$ is all real numbers x except $x = \pm 2$; the range of $f(x)$ is $(-\infty, 0] \cup (1, \infty)$
    (b) the y-intercept is $y = 0$; the x-intercept is $x = 0$
    (c) $y = 1$ is the horizontal asymptote
    (d) $x = \pm 2$ are the vertical asymptotes
    (e) there is no oblique asymptote

41. For the function $R(x) = \dfrac{3x}{x+4}$, the vertical asymptote is $x = -4$ and the horizontal asymptote is $y = 3$.

44. For the function $G(x) = \dfrac{-x^2+1}{x^2-5x+6}$, the vertical asymptotes are $x = 2,\ 3$ and the horizontal asymptote is $y = -1$.

51. First, rewrite the function $G(x) = \dfrac{x^3-1}{x-x^2}$ as:

$$G(x) = \frac{(x-1)(x^2+x+1)}{-x(x-1)}$$

We note that there is a hole at $x = 1$ since the multiplicity of 1 as a root of the numerator is the same as that of the denominator. Therefore, $x = 0$ is the only vertical asymptote.

Further rewriting $G(x)$, we have:

$$G(x) = -\frac{x^2+x+1}{x} = -x - 1 - \frac{1}{x}$$

Therefore, the oblique asymptote is $y = -x - 1$.