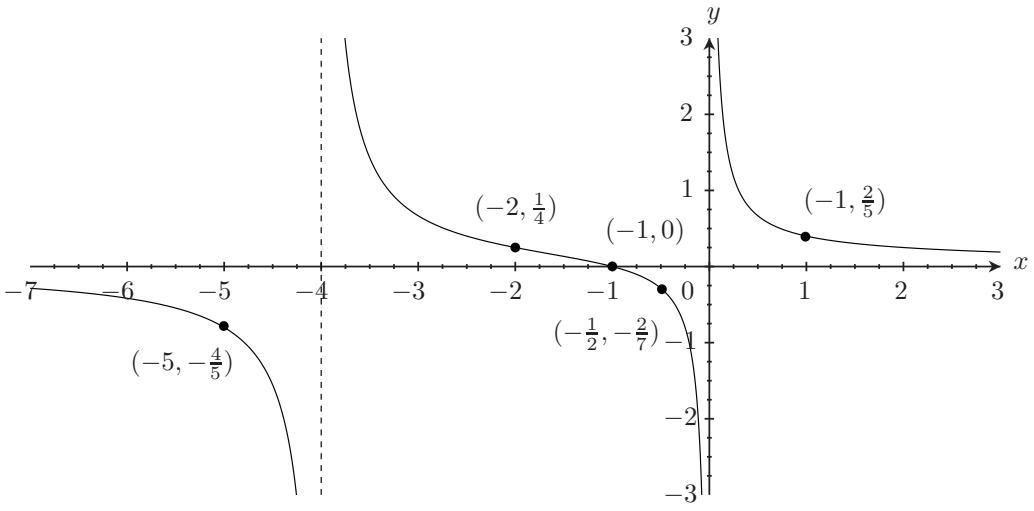
Math 121 – Section 4.3 Solutions

7. $R(x) = \dfrac{x+1}{x(x+4)}$

- the domain is all x except $x = 0,\ -4$; there is no y-intercept
- the x-intercept is at $x = -1$
- the vertical asymptotes are $x = 0$ and $x = -4$
- the horizontal asymptote is $y = 0$
- table:

Interval	$(-\infty, -4)$	$(-4, -1)$	$(-1, 0)$	$(0, \infty)$
Number Chosen	-5	-2	$-\frac{1}{2}$	1
Value of R	$R(-5) = -\frac{4}{5}$	$R(-2) = \frac{1}{4}$	$R(-\frac{1}{2}) = -\frac{2}{7}$	$R(1) = \frac{2}{5}$
Location of graph	below x-axis	above x-axis	below x-axis	above x-axis
Point on graph	$(-5, -\frac{4}{5})$	$(-2, \frac{1}{4})$	$(-\frac{1}{2}, -\frac{2}{7})$	$(1, \frac{2}{5})$

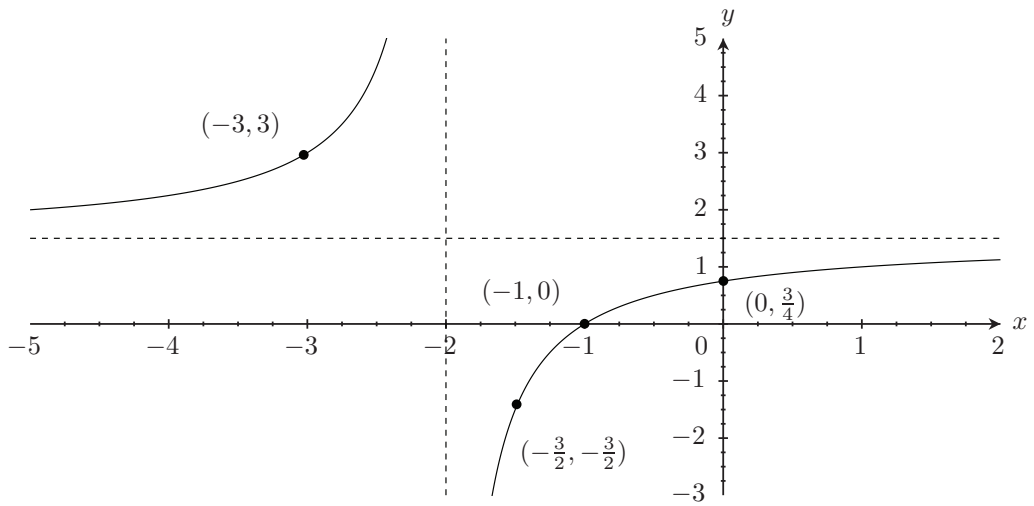


9. $R(x) = \dfrac{3x+3}{2x+4} = \dfrac{3(x+1)}{2(x+2)}$

- the domain is all x except $x = -2$; the y-intercept is at $R(0) = \frac{3}{4}$
- the x-intercept is at $x = -1$
- the vertical asymptote is $x = -2$
- the horizontal asymptote is $y = \frac{3}{2}$

- table:

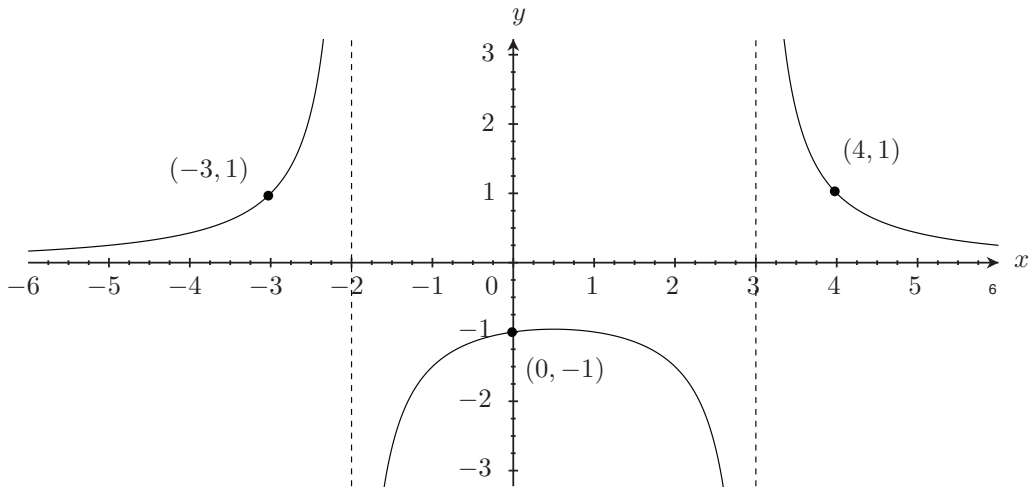
Interval	$(-\infty, -2)$	$(-2, -1)$	$(-1, \infty)$
Number Chosen	-3	$-\frac{3}{2}$	0
Value of R	$R(-3) = 3$	$R(-\frac{3}{2}) = -\frac{3}{2}$	$R(0) = \frac{3}{4}$
Location of graph	above x-axis	below x-axis	above x-axis
Point on graph	$(-3, 3)$	$(-\frac{3}{2}, -\frac{3}{2})$	$(0, \frac{3}{4})$



12. $R(x) = \dfrac{6}{x^2 - x - 6} = \dfrac{6}{(x-3)(x+2)}$

- the domain is all x except $x = -2,\ 3$; the y-intercept is at $R(0) = -1$
- there is no x-intercept
- the vertical asymptotes are $x = -2$ and $x = 3$
- the horizontal asymptote is $y = 0$
- table:

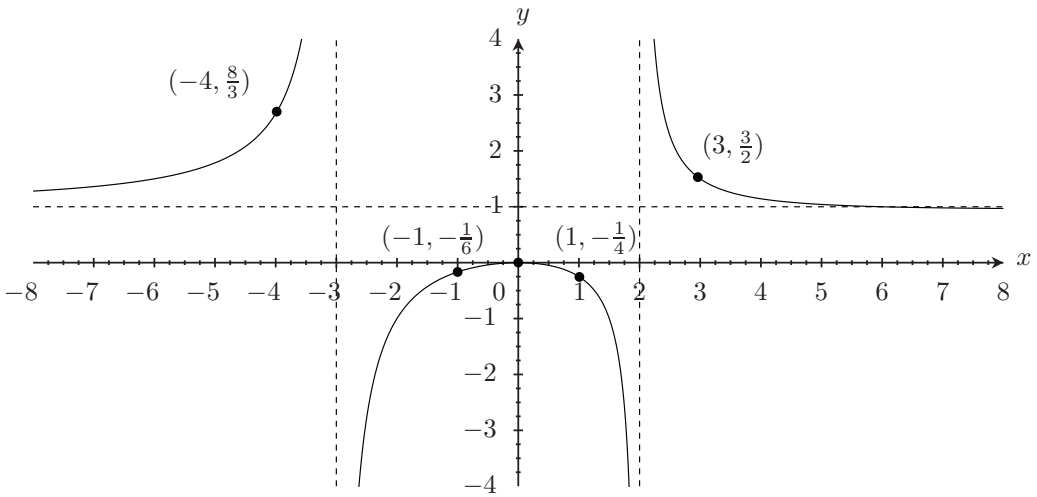
Interval	$(-\infty, -2)$	$(-2, 3)$	$(3, \infty)$
Number Chosen	-3	0	4
Value of R	$R(-3) = 1$	$R(0) = -1$	$R(4) = 1$
Location of graph	above x-axis	below x-axis	above x-axis
Point on graph	$(-3, 1)$	$(0, -1)$	$(4, 1)$



17. $R(x) = \dfrac{x^2}{x^2 + x - 6} = \dfrac{x^2}{(x+3)(x-2)}$

- the domain is all x except $x = -3,\ 2$; the y-intercept is at $R(0) = 0$
- the x-intercept is at $x = 0$
- the vertical asymptotes are $x = -3$ and $x = 2$
- the horizontal asymptote is $y = 1$
- table:

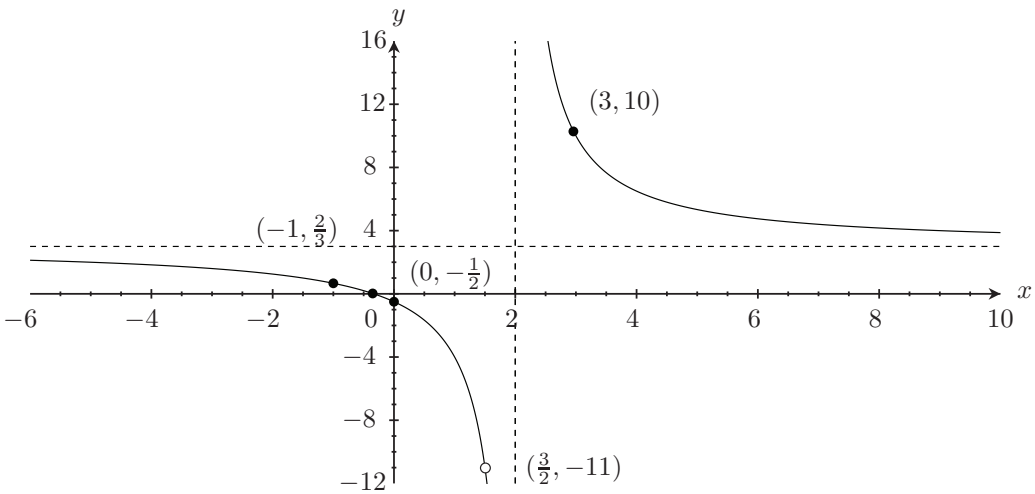
Interval	$(-\infty, -3)$	$(-3, 0)$	$(0, 2)$	$(2, \infty)$
Number Chosen	-4	-1	1	3
Value of R	$R(-4) = \frac{8}{3}$	$R(-1) = -\frac{1}{6}$	$R(1) = -\frac{1}{4}$	$R(3) = \frac{3}{2}$
Location of graph	above x-axis	below x-axis	below x-axis	above x-axis
Point on graph	$(-4, \frac{8}{3})$	$(-1, -\frac{1}{6})$	$(1, -\frac{1}{4})$	$(3, \frac{3}{2})$



35. $R(x) = \dfrac{6x^2 - 7x - 3}{2x^2 - 7x + 6} = \dfrac{(3x+1)(2x-3)}{(2x-3)(x-2)}$

- the domain is all x except $x = \frac{3}{2},\ 2$; the y-intercept is at $R(0) = -\frac{1}{2}$
- the x-intercept is at $x = -\frac{1}{3}$
- the vertical asymptote is $x = 2$
- the horizontal asymptote is $y = 3$
- there is a hole at $x = \frac{3}{2}$
- table:

Interval	$(-\infty, -\frac{1}{3})$	$(-\frac{1}{3}, 2)$	$(2, \infty)$
Number Chosen	-1	0	3
Value of R	$R(-1) = \frac{2}{3}$	$R(0) = -\frac{1}{2}$	$R(3) = 10$
Location of graph	above x-axis	below x-axis	above x-axis
Point on graph	$(-1, \frac{2}{3})$	$(0, -\frac{1}{2})$	$(3, 10)$



45. A rational function that might have the given graph is:

$$R(x) = \frac{x^2}{(x+2)(x-2)}$$

46. A rational function that might have the given graph is:

$$R(x) = -\frac{x}{(x+1)(x-1)}$$