Math 121 – Section 4.6 Solutions

7. $f(x)$ has degree 3 and its zeros are $3, 4-i$. Therefore,

$$f(x) = (x-3)[x-(4-i)][x-(4+i)]$$
$$f(x) = (x-3)(x^2-8x+17)$$
$$\boxed{f(x) = x^3 - 11x^2 + 41x - 51}$$

10. $f(x)$ has degree 4 and its zeros are $1, 2, 2+i$. Therefore,

$$f(x) = (x-1)(x-2)[x-(2+i)][x-(2-i)]$$
$$f(x) = (x^2-3x+2)(x^2-4x+5)$$
$$\boxed{f(x) = x^4 - 7x^3 + 19x - 23x + 10}$$

16. $f(x)$ has degree 6 and its zeros are $i, 3-2i, -2+i$. Therefore,

$$f(x) = (x-i)(x+i)[x-(3-2i)][x-(3+2i)][x-(-2+i)][x-(-2-i)]$$
$$f(x) = (x^2+1)(x^2-6x+13)(x^2+4x+5)$$
$$\boxed{f(x) = x^6 - 2x^5 - 5x^4 + 20x^3 + 59x^2 + 22x + 65}$$

17. $f(x)$ has degree 4 and its zeros are $3+2i; 4$, multiplicity 2. Therefore, one possible $f(x)$ is:

$$f(x) = [x-(3+2i)][x-(3-2i)](x-4)^2$$
$$f(x) = (x^2-6x+13)(x^2-8x+16)$$
$$\boxed{f(x) = x^4 - 14x^3 + 77x^2 - 200x + 208}$$

23. Given $f(x) = x^3 - 4x^2 + 4x - 16$ and $2i$ is a zero of $f(x)$, we have:

$$f(x) = (x-2i)(x+2i)g(x) = (x^2+4)g(x)$$

Using long division to find $g(x)$ we have:

$$\begin{array}{r|rrrr} & & x & & -4 \\ \hline x^2+4 & x^3 & -4x^2 & +4x & -16 \\ & -x^3 & & -4x & \\ \hline & & -4x^2 & & -16 \\ & & 4x^2 & & +16 \\ \hline & & & & 0 \end{array}$$

Therefore, the zeros of $f(x)$ are $\boxed{x = \pm 2i,\ 4}$.

26. Given $f(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18$ and $3i$ is a zero of $f(x)$, we have:

$$f(x) = (x - 3i)(x + 2i)q(x) = (x^2 + 9)q(x)$$

Using long division to find $g(x)$ we have:

$$\begin{array}{r|l}
 & 3x^2 \quad + 5x \quad - 2 \\
\hline
x^2 + 9 & 3x^4 + 5x^3 + 25x^2 + 45x - 18 \\
 & -\,3x^4 \qquad\quad - 27x^2 \\
\cline{2-2}
 & \qquad 5x^3 \; - 2x^2 + 45x \\
 & \quad\; - 5x^3 \qquad\quad - 45x \\
\cline{2-2}
 & \qquad\qquad - 2x^2 \qquad\quad - 18 \\
 & \qquad\qquad\;\; 2x^2 \qquad\quad + 18 \\
\cline{2-2}
 & \qquad\qquad\qquad\qquad\qquad 0
\end{array}$$

Since $3x^2 + 5x - 2 = (3x - 1)(x + 2)$, the real zeros of $f(x)$ are $x = \frac{1}{3}, -2$.

Therefore, the zeros of $f(x)$ are $\boxed{x = \pm 3i,\ \frac{1}{3},\ -2}$.

33. The potential rational real zeros of $f(x) = x^3 - 8x^2 + 25x - 26$ are $\pm 1, \pm 2, \pm 13, \pm 26$. Checking some of these we have:

$$\begin{aligned}
f(1) &= 1 - 8 + 25 - 26 = -8 \\
f(-1) &= -1 - 8 - 25 - 26 = -60 \\
f(2) &= 8 - 32 + 50 - 26 = 0
\end{aligned}$$

Therefore, $f(x)$ is factored as follows:

$$f(x) = (x - 2)q(x)$$

Using long division to find $q(x)$ we have:

$$\begin{array}{r|l}
 & x^2 \quad - 6x + 13 \\
\hline
x - 2 & x^3 - 8x^2 + 25x - 26 \\
 & -\,x^3 + 2x^2 \\
\cline{2-2}
 & \quad - 6x^2 + 25x \\
 & \qquad 6x^2 - 12x \\
\cline{2-2}
 & \qquad\qquad 13x - 26 \\
 & \qquad\quad - 13x + 26 \\
\cline{2-2}
 & \qquad\qquad\qquad\quad 0
\end{array}$$

Using the quadratic formula to solve $q(x) = x^2 - 6x + 13 = 0$ we have:

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)} \\
&= \frac{6 \pm \sqrt{36 - 52}}{2} \\
&= \frac{6 \pm 4i}{2} \\
&= 3 \pm 2i
\end{aligned}$$

The complex zeros of $f(x)$ are $\boxed{x = 3 \pm 2i}$.

36. The function $f(x) = x^4 + 13x^2 + 36$ is factored as follows:

$$f(x) = (x^2 + 4)(x^2 + 9)$$

Therefore, the complex zeros of $f(x)$ are $\boxed{x = \pm 2i,\ \pm 3i}$.