Math 121 – Section 5.1 Solutions

7. (a) $(f \circ g)(1) = f(g(1)) = f(0) = -1$
   (b) $(f \circ g)(-1) = f(g(-1)) = f(0) = -1$
   (c) $(g \circ f)(-1) = g(f(-1)) = g(-3) = 8$
   (d) $(g \circ f)(0) = g(f(0)) = g(-1) = 0$
   (e) $(g \circ g)(-2) = g(g(-2)) = g(3) = 8$
   (f) $(f \circ f)(-1) = f(f(-1)) = f(-3) = -7$

9. (a) $g(f(-1)) = g(1) = 4$
   (b) $g(f(0)) = g(0) = 5$
   (c) $f(g(-1)) = f(3) = -1$
   (d) $f(g(4)) = f(2) = -2$

12. $f(x) = 3x + 2$, $g(x) = 2x^2 - 1$
   (a) $(f \circ g)(4) = f(g(4)) = f(31) = 95$
   (b) $(g \circ f)(2) = g(f(2)) = g(8) = 127$
   (c) $(f \circ f)(1) = f(f(1)) = f(5) = 17$
   (d) $(g \circ g)(0) = g(g(0)) = g(-1) = 1$

17. $f(x) = |x|$, $g(x) = \dfrac{1}{x^2+1}$
   (a) $(f \circ g)(4) = f(g(4)) = f\left(\dfrac{1}{17}\right) = \dfrac{1}{17}$
   (b) $(g \circ f)(2) = g(f(2)) = g(2) = \dfrac{1}{5}$
   (c) $(f \circ f)(1) = f(f(1)) = f(1) = 1$
   (d) $(g \circ g)(0) = g(g(0)) = g(1) = \dfrac{1}{2}$

21. Given $f(x) = \dfrac{3}{x-1}$ and $g(x) = \dfrac{2}{x}$, the composite function $f \circ g$ is:

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f\left(\frac{2}{x}\right) \\ &= \frac{3}{\frac{2}{x} - 1} \\ &= \frac{3x}{2-x}\end{aligned}$$

The domain of $g(x)$ is all x except $x = 0$. The domain of $f(g(x))$ is all x except $x = 2$. Therefore, the domain of $f \circ g$ is $\boxed{\text{all } x \text{ except } x = 0,\ 2}$.

26. Given $f(x) = x - 2$ and $g(x) = \sqrt{1-x}$, the composite function $f \circ g$ is:

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(\sqrt{1-x}) \\ &= \sqrt{1-x} - 2\end{aligned}$$

The domains of both $g(x)$ and $f(g(x))$ are $x \leq 1$. Therefore, the domain of $f \circ g$ is $\boxed{x \leq 1}$.

35. $f(x) = \dfrac{3}{x-1}$ and $g(x) = \frac{2}{x}$

(a) The domain of $f \circ g$ is all x except $x = 0,\ 2$ (from Problem 21).

(b) The composite function $g \circ f$ is:

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g\left(\frac{3}{x-1}\right) \\ &= \frac{2}{\frac{3}{x-1}} \\ &= \frac{2}{3}(x-1)\end{aligned}$$

The domain of $f(x)$ is all x except $x = 1$. The domain of $g(f(x))$ is all x. Therefore, the domain of $g \circ f$ is $\boxed{\text{all } x \text{ except } x = 1}$.

(c) The composite function $f \circ f$ is:

$$\begin{aligned}(f \circ f)(x) &= f(f(x)) \\ &= f\left(\frac{3}{x-1}\right) \\ &= \frac{3}{\frac{3}{x-1} - 1} \\ &= \frac{3(x-1)}{3-(x-1)} \\ &= \frac{3(x-1)}{4-x}\end{aligned}$$

The domain of $f(x)$ is all x except $x = 1$. The domain of $f(f(x))$ is all x except $x = 4$. Therefore, the domain of $f \circ f$ is $\boxed{\text{all } x \text{ except } x = 1,\ 4}$.

(d) The composite function $g \circ g$ is:

$$\begin{aligned}(g \circ g)(x) &= g(g(x)) \\ &= g\left(\frac{2}{x}\right) \\ &= \frac{2}{\frac{2}{x}} \\ &= x\end{aligned}$$

The domain of $g(x)$ is all x except $x = 0$. The domain of $g(g(x))$ is all x. Therefore, the domain of $g \circ g$ is $\boxed{\text{all } x \text{ except } x = 0}$.

45. $f(x) = 2x$ and $g(x) = \frac{1}{2}x$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f\left(\frac{1}{2}x\right) \\ &= 2\left(\frac{1}{2}x\right) \\ &= x \\ (g \circ f)(x) &= g(f(x)) \\ &= g(2x) \\ &= \frac{1}{2}(2x) \\ &= x\end{aligned}$$

53. Two possible functions f and g that satisfy:

$$f \circ g = (2x+3)^4$$

are $\boxed{f(x) = x^4 \text{ and } g(x) = 2x+3}$.