Math 121 – Section 5.8 Solutions

1. The size P of a certain insect population at time t (in days) obeys the function $P(t) = 500e^{0.02t}$.

   (a) $P(0) = 500$

   (b) The growth rate is 0.02.

   (c) $P(10) = 500e^{0.02(10)} = 500e^{0.2} \approx 610$

   (d) The time t when $P(t) = 800$ is:

$$\begin{aligned} 800 &= e^{0.02t} \\ 0.02t &= \ln 800 \\ \boxed{t = \frac{\ln 800}{0.02} \approx 334.23 \text{ days}} \end{aligned}$$

   (e) The time t when $P(t) = 1000$ is:

$$\boxed{t = \frac{\ln 1000}{0.02} \approx 345.39 \text{ days}}$$

4. Iodine 131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in days). Assume that a scientist has a sample of 100 grams of iodine 131.

   (a) The decay rate is -0.087.

   (b) $A(9) = 100e^{-0.087(9)} \approx 45.70$ grams

   (c) The time t when $A(t) = 70$ is:

$$\begin{aligned} 70 &= 100e^{-0.087t} \\ -0.087t &= \ln \frac{70}{100} \\ \boxed{t = \frac{\ln \frac{70}{100}}{-0.087} \approx 4.1 \text{ days}} \end{aligned}$$

   (d) The time t when $A(t) = 50$ is:

$$\boxed{t = \frac{\ln \frac{50}{100}}{-0.087} \approx 7.97 \text{ days}}$$

8. The population of a midwestern city follows the exponential law.

   (a) If N is the population of the city and t is the time in years, then:

$$N(t) = N_0 e^{kt}$$

(b) If the population decreased from 900,000 to 800,000 from 2003 to 2005 then:

$$N(0) = 900,000,\ N(2) = 800,000$$

Therefore, solving for k we have:

$$\begin{aligned} N(2) &= N_0 e^{2k} \\ 800,000 &= 900,000 e^{2k} \\ \frac{8}{9} &= e^{2k} \\ 2k &= \ln \frac{8}{9} \\ k &= \frac{\ln \frac{8}{9}}{2} \approx -0.059 \end{aligned}$$

Using the value of k above, the population in 2007 is:

$$\boxed{N(4) = 900,000 e^{4k} \approx 711,111}$$

14. A thermometer reading 72° F is placed in a refrigerator where the temperature is a constant 38° F.

(a) Using Newton's Law of Cooling:

$$u(t) = T + (u_0 - T)e^{-kt}$$

we know that $T = 38$ and $u_0 = 72$. If the thermometer reads 60° F after 2 minutes, the value of k is:

$$\begin{aligned} 60 &= 38 + (72 - 38)e^{-2k} \\ e^{-2k} &= \frac{60 - 38}{72 - 38} \\ e^{-2k} &= \frac{22}{34} \\ -2k &= \ln \frac{22}{34} \\ k &= -\frac{\ln \frac{22}{34}}{2} \approx 0.22 \end{aligned}$$

After 7 minutes, the thermometer reads:

$$\boxed{u(7) = 38 + (72 - 38)e^{-7k} \approx 45.41^\circ \text{ F}}$$

(b) The time t when the thermometer reads 39° F is:

$$\begin{aligned} 39 &= 38 + (72 - 38)e^{-kt} \\ 1 &= 34e^{-kt} \\ e^{-kt} &= \frac{1}{34} \\ -kt &= \ln \frac{1}{34} \\ \end{aligned}$$

$$\boxed{t = -\frac{\ln \frac{1}{34}}{k} \approx 16.2 \text{ minutes}}$$

(c) The time t when the thermometer reads 45° F is:

$$
\begin{aligned}
45 &= 38 + (72 - 38)e^{-kt} \\
7 &= 34e^{-kt}
\end{aligned}
$$

$$\boxed{t = -\frac{\ln \frac{7}{34}}{k} \approx 7.18 \text{ minutes}}$$

(d) As time passes, the temperature decreases and approaches $T = 38°$ F.

23. The logistic growth model

$$P(t) = \frac{0.9}{1 + 6e^{-0.32t}}$$

relates the proportion of U.S. households that own a DVD player to the year. Let $t = 0$ represent 2000, $t = 1$ represent 2001, and so on.

(a) The maximum proportion of households that will own a DVD player is 0.9.

(b) $P(0) = \dfrac{0.9}{1+6} \approx 0.13$

(c) $P(5) = \dfrac{0.9}{1 + 6e^{-0.32(5)}} \approx 0.41$

(d) The time t when 80% of households own a DVD player is:

$$
\begin{aligned}
0.8 &= \frac{0.9}{1 + 6e^{-0.32t}} \\
1 + 6e^{-0.32t} &= \frac{0.9}{0.8} \\
e^{-0.32t} &= \frac{\frac{0.9}{0.8} - 1}{6}
\end{aligned}
$$

$$\boxed{t = -\frac{\ln\left(\frac{\frac{0.9}{0.8}-1}{6}\right)}{0.32} \approx 12.1 \text{ years}}$$